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THE COMMITMENT VALUE OF FUNDING PENSIONS

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The commitment value of funding pensions

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Abstract

This paper studies how funding public pensions can improve policy outcomes when short-sighted governments cannot commit. We focus on sustainable plans, where optimal nonlinear pensions are not reneged on by sequential governments. Funding pensions is a commitment mechanism. It implies lower contributions than does the second best policy, which reduces temptation to over-redistribute later and to misuse revealed private information. Funding may be preferable even if the population growth rate is higher than the rate of return on assets. Second best optimal policies are also more likely to be renegotiation proof under fully funded pensions.

Keywords: Pensions, Commitment, Redistribution, Funding

JEL classification: H55, H31

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1 Introduction

Publicly managed pension plans are subjected to political risks (Diamond, 1994, 1996). Governments, even if they are benevolent, may be tempted to engage in excess redistribution across retirees using pension wealth. Because of this, some have argued that funding and privatizing public pensions could reduce political risks

Recent literature in dynamic optimal taxation, among which Farhi et al. (2012), has shown that commitment is especially relevant in non-linear optimal tax problems, in which the fiscal schedule must induce individuals to reveal private information about themselves. If the policy maker can improperly use revealed information and renege on its promises, the characteristics of the optimal policy may be significantly affected.

Farhi et al. (2012) show that governments who cannot commit should tax capital accumulation progressively. This has the effect of reducing income inequality in the optimum. Sequentially, governments thus have fewer incentives too misuse households' private information to over-redistribute. They study sustainable equilibria à la Chari and Kehoe (1990) that are perfect Bayesian and that can be sustained by a trigger-type reaction by the households following a governmental deviation.

In a simpler framework, we extend their analysis to show how the institutional structure of public pension, whether it is fully funded or unfunded, may help or harm pension policy outcomes when commitment is assumed away. We use a simple, overlapping generations model, with infinite repeated between governments and successive generations of individuals. An initial social planner who sets the overall generosity and the redistributive characteristics of a pension plan must ensure that successive short-sighted governments do not have an incentive to renege on the initial promises.

Our results formalize the idea that funding pensions may be used as a commitment mechanism. When they are, the optimal response to a lack of commitment is to reduce aggregate pension contributions to reduce next period's temptation. With unfunded plans, immediate temptation to over redistribute involves higher contributions than in the second best plan and significantly less inequality. We use numerical examples to show that optimal second best policies are more likely to be sustainable under funded pensions. Due to its pre-commitment value, funded pensions may be preferable to pay-as-you-go schemes even when population grows faster than the rate of return on financial assets.

2 Model

Consider an overlapping generations version of Stiglitz (1982) where individuals live for two periods of equal duration. In the first half of their lives individuals supply labor, consume, are taxed and contribute to a public pension fund. In their second half they are retired and live off public pension benefits. The timing of retirement is exogenous and population grows at a fixed rate $\eta > 0$. Thus, at each period $t = 0, 1, ...$ one generation of workers cohabits with one generation of retirees. Therefore, the constant ratio of workers to retirees is $1 + \eta$. There is a constant proportion n_i of type$i$ agents, where types are denoted by $i = 1, 2$. There is an underlying linear production technology according to which a type$i$ worker who supplies ℓ_t^i units of labor faces a hourly market wage rate w_i with $w_1 < w_2$. Gross incomes are defined as $y_t^i \equiv w_i \ell_t^i$. All individuals have identical, time separable utility functions:

$$U(c_t^i, \ell_t^i, d_{t+1}^i) = u(c_t^i) - z(y_t^i / w_i) + \beta u(d_{t+1}^i) \tag{1}$$

where c_t^i, $y_t^i / w_i \equiv \ell_t^i$ and d_{t+1}^i are respectively the consumption level of a worker born at t, the worker's labor supply and the worker's consumption when old at $t + 1$. Instantaneous consumption utility u is strictly increasing, strictly concave and obey the limiting condition $u'(0) = \infty$. The utility cost of supplying labor (z) is strictly increasing and strictly convex with $z'(0) = 0$ and $z''(\ell) > 0, \forall \ell$. The utility function satisfies the single-crossing condition since, the marginal cost of earning gross revenue satisfies $z'(y)/w_2 < z'(y)/w_1 \ \forall y$.

A social planner ranks allocations $\phi_t \equiv \{c_t^i, y_t^i, d_t^i\}_{i=1}^2, \forall t$ using a welfarist social welfare function:

$$W_0 = \sum_{t=0}^{\infty} \delta^t \left(\sum_{i=1}^{2} n_i \left[u(c_t^i) - z(y_t^i/w_i) + \beta u(d_{t+1}^i) \right] \right) \tag{2}$$

where $\delta = (1+\eta)/(1+\rho)$ is the inter-generational discount factor, and $\rho > \eta$ is the inter-generational discount rate. We emphasize the effects of (un)-funding pensions on optimal policies by writing the feasibility constraints as the following:

$$\sum_i n_i c_t^i = \sum_i n_i y_t^i - b_t^i \tag{3a}$$

$$\sum_i n_i d_t^i = (1-\alpha)(1+\eta)b_t + \alpha(1+r)b_{t-1}. \tag{3b}$$

where b_t is the aggregate pension contributions of the generation born at t. By (3a) aggregate consumption of workers equals aggregate gross income minus pension contributions. By (3b) aggregate consumption of retirees depends on $\alpha \in \{0, 1\}$, which captures whether public pensions are unfunded fully funded. With $\alpha = 1$ consumption of current retirees is funded through their own past contributions. For simplicity, assume that these savings yield the fixed rate of return r, as we would find in a small open economy. With $\alpha = 0$ pensions are paid with current contributions. [Moreover, our modelling implicitly means that funding pensions ex ante shuts down all inter-generational redistribution. This only serves illustrative purposes.

It is worth noting that α is taken as an institutional feature that would be highly costly to reform at short notice. This assumption captures the stylized fact that pension contribution rates are more frequently adjust than the fundamental structure of public pension funds, which requires in-depth reform. Moreover, ruling out intermediary cases $0 < \alpha < 1$ discards issues of convergence and allows us to directly analyze steady states, without overshadowing the intuition this paper seeks to convey.

2.1 Full commitment benchmark

Choosing an optimal allocation is equivalent to designing a nonlinear tax system across workers and retirees. Suppose that at $t = 0$ the social planner can once and for all promise future allocations that satisfy the feasibility constraints. He maximizes (2) by choosing ϕ_t, $\forall t$ subject to (3a) and (3b).[1] Unsurprisingly, concave utility of consumption (or aversion to inequality) gives $c_t^1 = c_t^2$, $d_t^1 = d_t^2$, and $y_t^1 < y_t^2$ $\forall t$. All individuals have identical consumptions, but type2s are invited to work more (Mirrlees, 1971; Stiglitz, 1982).

As is well known since Mirrlees (1971), such an allocation is not incentive compatible. If only gross incomes y_t^i can be observed instead of types, type2 workers would mimic type1s. Second best optimality is therefore restricted to incentive compatible allocations that satisfy

$$u(c_t^1) - z(y_t^1/w_2) + \beta u(d_{t+1}^1) \leq u(c_t^2) - z(y_t^2/w_2) + \beta u(d_{t+1}^2). \tag{4}$$

Full commitment implies that the social planner commits to allocations before private information is revealed by households. The second best allocation satisfies $c_t^1 = d_t^1 < d_t^2 = c_t^2$ with $y_t^1 < y_t^2$. The fact that interests us the most is that consumption smoothing is preserved:

$$\begin{cases} u'(c_t^i)/u'(d_{t+1}^i) = \beta(1+r) & \text{if } \alpha = 1 \\ u'(c_t^i)/u'(d_t^i) \quad\; = \beta(1+\rho) & \text{if } \alpha = 0. \end{cases} \tag{5}$$

2.2 Sequential governments

Suppose now that the *social planner* initially promises allocations $\phi_t, \forall t$. Each allocation must be incentive compatible and feasible. Lagrange multipliers θ_t, μ_t, and λ_t are assigned to equations (3a), (3b) and (4). However, the social planner does not have the final say. Sequential governments can later re-optimize and change allocations insofar as they are

[1] The first-order conditions of all Lagrangian problems are produced in the appendix.

feasible. We model them in the spirit of Farhi et al. (2012), where three motives induce sequential governments to renege. First, they already know retirees' types and may seek to set $d_t^1 = d_t^2$. Second, they may weigh generations differently than does the initial planner. Third, accumulated assets are perceived as an inelastic tax base that can be redistributed at no immediate efficiency cost. The objective function of a time t government is

$$W_t = \pi\beta u(d_t) + (1-\pi)\sum_i n_i[u(c_t^i) - z(y_t^i/w_i) + \beta u(d_{t+1})] \tag{6}$$

where π is the weight put on current retirees whose types are known.

Let us focus on allocations that can be promised by the planner at $t = 0$ and which sequential governments will not renege on. Oftentimes, such policies have been characterized by taking the limit of the backward induction solution to a dynamic game.[2] Here, young workers are conscious that their private information will be used to equalize consumption across retirees. Therefore, they reveal it only if the promised allocations maximize (6) subject to $d_t^1 = d_t^2 = d_t$, to the feasibility constraint and to the IC constraint.

To separate types, allocations must therefore allow for more inequality across workers. But what interests us is the role of accumulated assets b_t on the outcome of this game. Denoting $\bar{\sigma}_t$ the allocation selected by governments (and promised by the social planner) and by $\overline{W}_t$ a government's value function, we find that

$$\partial\overline{W}_t/\partial b_{t-1} = \alpha(1-\pi)\beta u'(b_{t-1}(1+r))(1+r). \tag{7}$$

Thus, increasing time t contributions increases next year's government's utility only if pensions are funded. Then, current governments can choose future governments' cash-on-hand. With unfunded pensions, current governments simultaneously choose current contributions and current retirees' consumptions, when temptation to put too much weight on the retirees

[2] For instance, see Boadway et al. (1996b,a); Boadway and Keen (1998); Brett and Weymark (2008); Debortoli and Nunes (2010); Krause and Guo (2011a,b).

is maximal.

2.3 Sustainable contribution rules

As shown by Chari and Kehoe (1990), a better social outcome can be sustained if households' decisions depend on history, and if history is used to "punish" governments using a trigger-type strategy (the first-order conditions are provided in the appendix). History h_{t-1} consists of all the allocations that have been implemented in the past. Sequential governments' strategies $\phi_t(h_{t-1})$ depend on history as well.

For households' behavior, we focus on symmetric strategies (no coordination) and seek for a Perfect Bayesian Equilibrium. Sequential rationality requires that sustainable allocations are not sustained by punishment strategy that are not subgame perfect (such as households threatening never to work and consume again). The best sequence of allocations that can be sustained without commitment is such that households believe that a sequential government will not break the social planner's promises if none have been broken up until now. Whenever one government has reneged, households revert to a strategy in which they believe that governments will always be shortsighted and seek to obtain $\overline{W}_t(\alpha b_{t-1})$. This is the harshest subgame perfect trigger strategy that gives us the sustainable policy that is closest to the second best.

Given this trigger strategy, sequential governments prefer the allocation promised by the social planner to re-optimizing and get $\overline{W}_t(\alpha b_{t-1})$. Farhi et al. (2012) have shown that, as a consequence, the social planners' promises must comply with sequence of credibility constraints, which are

$$\pi \sum_i n_i \beta u(d_t^i) + (1-\pi) \sum_i n_i [u(c_t^i) - z(y_t^i/w_i) + \beta u(d_{t+1}^i)] \geq \overline{W}_t(\alpha b_{t-1}), \quad \forall t. \tag{8}$$

Whenever a standard second best allocation is not sustainable, (8) is binding at t in the social planner's maximization problem. The left-hand side shows a government's welfare under the initially promised allocation. The right-hand side gives its welfare if it exploits retirees' information, and over redistributes, but where the economy reverts to the static outcome forever.

From (8) it is now apparent how sustainability requirements will modify the initial allocation. With unfunded pensions, sequential governments can yield to temptation and immediately increase resources available for retirees. Therefore, the solution is for the social planner to partially "yield to temptation" ex ante. With funded pensions, sequential governments can reduce contributions instead, so as to cut immediate resources to sequential governments. This is so because $\partial\overline{W}_{t+1}/\partial b_t > 0$ if and only if $\alpha = 1$. Thus, instead of giving sequential governments what they want, it can worsen the penalty suffered by the economy if any government ever reneges.

To show this clearly, let us derive the analogous contribution rules to (5) but for the best possible sustainable policy. The social planner chooses $\phi_t, \forall t$ by maximizing (2) subject to the IC constraint (4) to which we assign Lagrange multipliers μ_t, to the resource constraints (3a) and (3b) with multipliers λ_t and θ_t and to the credibility constraint with multiplier γ_t.

For type1 individuals, contributions with unfunded pensions satisfies (the intuition is identical for type2s, as discussed in the appendix):

$$\left(\frac{n_1 - \mu_t + \gamma_t n_1(1-\pi)}{n_1 - \mu_{t-1} + \delta\gamma_t n_1 \pi + \gamma_{t-1} n_1(1-\pi)}\right)\frac{u'(c_t^1)}{u'(d_t^1)} = \beta(1+\rho) \tag{9}$$

Equation (9) shows that when the credibility constraint binds every period sets $c_t < d_t$ for both types and increase contributions. This starkly contrasts with the same conditions under funding, which is the following:

$$\left(\frac{n_1 - \mu_t + \gamma_t n_1(1-\pi)}{n_1 - \mu_t + \delta\gamma_{t+1} n_1 \pi + \gamma_t n_1(1-\pi)}\right)\frac{u'(c_t^1)}{u'(d_{t+1}^1)} = \beta(1+r) - (1-\pi)\frac{\gamma_{t+1}}{\theta_{t+1}}\beta u'(\bar{d}_t). \tag{10}$$

The added term on the right-hand side captures the commitment value of funded pensions. By reducing aggregate contributions at t, the social planner reduces cash-on-hand available to the government at $t = 1$. As a consequence, it reduces temptation one period ahead and makes the next period's credibility constraint less binding. Thus, we should expect that funding allows to sustain a policy closer to the second best.

2.4 Numerical illustrations

We illustrate the potential benefits of funding pensions using two numerical examples. We use the utility function $u(c) = c^{1-\rho}/(1-\rho)$ with $\rho = 0.85$ and quadratic disutility of labor $z(y/w) = \sigma(y/w)^2/2$. We fix the inter-temporal discount factor at $\delta = 0.995$. A high value of δ implies that social welfare under full commitment will be similar with funded and unfunded pensions when $r = \eta$. The two illustrative scenarios are reported in table 1.

We first study a scenario where $r = \eta = 1$. If π is low enough so the credibility constraint does not bind at the second best allocation, the funded and unfunded regimes yield a social welfare of approximately 9.6321 in the steady state. Inter generational discounting implies that retirees' consumption is marginally higher under unfunded provision.

When pensions are unfunded, increases in π quickly translate into binding temptation. Since no commitment mechanism is available, sequential governments react by increasing retirees' consumption. Otherwise, reneging would take place later on. Social welfare decreases to eventually attain negative values.

With funding, a broader range of second best allocations can be sustained without commitment. Only when π passes from 0.60 to 0.65 does the credibility constraint binds. When it does, one can readily see how pre-commitment kicks in. Instead of increasing contributions (as under unfunded pensions), the social planner reduces them to diminish sequential governments' amount of cash-on-hand that is available for redistribution.

The second numerical example is found in the four rightmost columns of table 1. We have set $\eta > r$ and adjusted ρ so that δ remains unchanged. In our setup, $\eta > r$ makes the unfunded regime strictly dominant in any full commitment scenario. However, as π increases the commitment value of funding pensions makes it preferable not to resort to pay-as-you-go schemes.

	$r = \eta = 1, \delta = 0.995$				$r = 0.5,\ \eta = 1.5,\ \delta = 0.995$			
	Unfunded		Funded		Unfunded		Funded	
	W_0	b_t	W_0	b_t	W_0	b_t	W_0	b_t
π								
0.35	9.6321	1.5244	9.6321	1.5184	9.7506	1.5742	9.6321	1.5184
0.40	9.6282	1.6757	9.6321	1.5184	9.7466	1.7277	9.6321	1.5184
0.45	9.6036	1.9550	9.6321	1.5184	9.7214	2.0133	9.6321	1.5184
0.50	9.5546	2.2620	9.6321	1.5184	9.6713	2.3262	9.6321	1.5184
0.55	9.4770	2.6023	9.6321	1.5184	9.5919	2.6725	9.6321	1.5184
0.60	9.3634	2.9845	9.6321	1.5184	9.4754	3.0608	9.6321	1.5184
0.65	9.2010	3.4214	9.6318	1.4824	9.3090	3.5041	9.6318	1.4829
0.70	8.9675	3.9337	9.6306	1.4453	9.0694	4.0232	9.6306	1.4457
0.75	8.6207	4.5561	9.6285	1.4086	8.7130	4.6539	9.6286	1.4090
0.80	8.0576	5.3536	9.6256	1.3721	8.1480	5.4622	9.6257	1.3724
0.90	4.9948	8.2718	9.6166	1.2967	4.9644	8.4260	9.6166	1.2969
0.940	0.2050	11.1691	9.6110	1.2631	-0.0157	11.3796	9.6111	1.2634
0.945	-0.9958	11.7659	9.6102	1.2586	-1.2688	11.9901	9.6126	1.2587
0.950	-2.5048	12.4703	9.6094	1.2539	-2.8460	12.7118	9.6094	1.2541

3 Conclusion

We used a simple model to formalize why funding pensions may help governments to commit. Our stylized assumptions helped us formalize why funding may act as a commitment device. The funding structure of the pension plan is taken as given, and funding can completely shut down the inter-generational redistribution mechanism.

Of course, more has still to be done on this topic. Some researchers, such as Blake (2000) and Barr (2002), contend that funded pensions is at best an imperfect commitment device to isolate pension capital from political risks.[3] While governments can (and do) break their PAYG promises, they can equally reduce the real return to pension funds, by requiring fund managers to hold government financial assets with a lower yield than they could earn elsewhere, or by withdrawing or reducing any tax privileges. The Argentinean case also convincingly demonstrates that simply ending pay-as-you-go schemes and transferring pension management to the private sector does not mechanically alleviate political risks (Kay, 2009). A new set of political risks can then emerge since funded assets can be perceived as an inelastic tax base by predatory and short-sighted governments, with excess redistribution and time-inconsistent policy-making as consequences.

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A Supplementary material

A.1 Sequential government's problem under deviation

The Lagrangian of a sequential government that characterizes a history independent policy) is

$$
\begin{aligned}
\overline{\mathcal{L}}_t = {} & \pi\beta u(d_t) + (1-\pi)\sum_i n_i[u(c_t^i) - z(y_t^i/w_i)] + (1-\pi)\beta u(d_{t+1}). \\
& - \lambda_t\left[\sum_i n_i c_t^i - \sum_i n_i y_t^i + b_t\right] \\
& - \mu_t\left[u(c_t^1) - z(y_t^1/w_2) - u(c_t^2) + z(y_t^2/w_2)\right].
\end{aligned} \tag{11}
$$

Use the fact that $d_{t+1} = \alpha b_t(1+r) + (1-\alpha)d_{t+1}(1+\eta)$. The government only chooses contemporaneous choice variables, which yields the following first-order conditions:

$$c_t^1 : ((1-\pi)n_1 - \mu_t)u'(c_t^1) - n_1\lambda_t \qquad = 0 \tag{12a}$$

$$c_t^2 : ((1-\pi)n_2 + \mu_t)u'(c_t^2) - n_2\lambda_t \qquad = 0 \tag{12b}$$

$$d_t : \pi\beta u'(d_t) - \theta_t \qquad = 0 \tag{12c}$$

$$y_t^1 : -n_1(1-\pi)z'(y_t^1/w_1)/w_1 + \mu_t z'(y_t^1/w_2)/w_2 + n_1\lambda_t \qquad = 0 \tag{12d}$$

$$y_t^2 : -(n_2(1-\pi) + \mu_t)z'(y_t^2/w_2)/w_2 + n_2\lambda_t \qquad = 0 \tag{12e}$$

$$b_t : -\lambda_t + (1-\alpha)\pi u'(d_t)(1+\eta) + \alpha(1-\pi)u'(d_{t+1}) \qquad = 0. \tag{12f}$$

The ensuing value function $\overline{W}_t(\alpha b_{t-1})$ satisfies $\partial\overline{W}_t/\partial b_{t-1} = (1-\pi)\alpha\beta u'(b_{t-1})$, which is positive if and only if $\alpha = 1$, since b_{t-1} has been chosen last period.

A.2 Social planner' problems

The Lagrangian of the social planner is

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \delta^t \Bigg\{ & \sum_i \left[u(c_t^i) - z(y_t^i/w_i) + \beta u(d_{t+1}^i)\right] \\
& - \lambda_t \left[\sum_i n_i c_t^i - \sum_i n_i y_t^i + b_t\right] \\
& - \theta_t \left[\sum_i n_i d_t^i - (1-\alpha)(1+\eta) b_t - \alpha(1+r) b_{t-1}\right] \\
& - \mu_t \left[u(c_t^1) - z(y_t^1/w_2) + \beta u(d_{t+1}^1) - u(c_t^2) + z(y_t^2/w_2) - \beta u(d_{t+1}^2)\right] \\
& - \gamma_t \left[\widehat{W}_t(\alpha b_{t-1}) - \pi \sum_i n_i \beta u(d_t^i) - (1-\pi) \sum_i n_i [u(c_t^i) - z(y_t^i/w_i) + \beta u(d_{t+1}^i)]\right] \Bigg\}.
\end{aligned}
\tag{13}
$$

Using $\partial W_t(\alpha b_t)/\partial b_t = \alpha(1-\pi)\beta u'(\bar{d}_{t+1})\alpha(1+r)$ the first-order conditions are, $\forall t:$

$$c_t^1 : \delta^t(n_1 - \mu_t)u'(c_t^1) - \delta^t n_1 \lambda_t + \delta^t \gamma_t n_1 (1-\pi) u'(c_t^1) \qquad = 0 \tag{14a}$$

$$c_t^2 : \delta^t(n_2 + \mu_t)u'(c_t^2) - \delta^t n_2 \lambda_t + \delta^t \gamma_t n_2 (1-\pi) u'(c_t^2) \qquad = 0 \tag{14b}$$

$$d_t^1 : \delta^{t-1}(n_1 - \mu_{t-1})\beta u'(d_t^1) - \delta^t \theta_t n_1 + \gamma_t \delta^t \pi n_1 \beta u'(d_t^1) + \gamma_{t-1}\delta^{t-1}(1-\pi) n_1 \beta u'(d_t^1) \qquad = 0 \tag{14c}$$

$$d_t^2 : \delta^{t-1}(n_2 + \mu_{t-1})\beta u'(d_t^1) - \delta^t \theta_t n_2 + \gamma_t \delta^t \pi n_2 \beta u'(d_t^2) + \gamma_{t-1}\delta^{t-1}(1-\pi) n_1 \beta u'(d_t^2) \qquad = 0 \tag{14d}$$

$$y_t^1 : -\delta^t n_1 z'(y_t^1/w_1)/w_1 + \delta^t \mu_t z'(y_t^1/w_2)/w_2 + \delta^t n_1 \lambda_t + \delta^t \gamma_t (1-\pi) z'(y_t^1/w_2)/w_2 \qquad = 0 \tag{14e}$$

$$y_t^2 : -\delta^t (n_2 + \mu_t) z'(y_t^2/w_2)/w_2 + \delta^t n_2 \lambda_t + \delta^t \gamma_t (1-\pi) z'(y_t^2/w_2)/w_2 \qquad = 0 \tag{14f}$$

$$b_t : -\delta^t\lambda_t + \delta^t\theta_t(1-\alpha)(1+\eta) + \delta^{t+1}\theta_{t+1}\alpha(1+r) - \delta^{t+1}(1-\pi)\gamma_{t+1}\beta u'(\bar{d}_t)\alpha(1+r) \quad = 0 \tag{14g}$$

In the first best we impose $\mu_t = 0$ and $\gamma_t = 0$. The first-order conditions reduce to. In the second best with commitment, we impose $\mu_t > 0$ and $\gamma_t = 0$. The conditions the give. Finally, without commitment we let $\mu_t > 0$ and let $\gamma_t \geq 0$ depending on whether the second best is sustainable.

<u>Unfunded case</u>

Using (14a) and (14c) and simplifying for δs gives

$$\left(\frac{n_1 - \mu_t + \gamma_t n_1(1-\pi)}{n_1 - \mu_{t-1} + \gamma_{t-1}n_1(1-\pi) + \delta\gamma_t n_1 \pi}\right)\frac{u'(c_t^1)}{u'(d_t^1)} = \frac{\beta}{\delta}\frac{\lambda_t}{\theta_t}.$$

By (14g), $\lambda_t/\theta_t = 1+\eta$ and the definition of δ

$$\left(\frac{n_1 - \mu_t + \gamma_t n_1(1-\pi)}{n_1 - \mu_{t-1} + \delta\gamma_t\pi n_1 + \gamma_{t-1}(1-\pi)n_1}\right)\frac{u'(c_t^1)}{u'(d_t^1)} = \beta(1+\rho).$$

A similar operation is conducted with type2 individuals (except that $n_1 - \mu_t$ is replaced by $n_2 + \mu_t$).

<u>Fully funded case</u>

Again doing the analysis for type1 individuals, (14a) and (14c) gives

$$\left(\frac{n_1 - \mu_t + \gamma_t n_1(1-\pi)}{n_1 - \mu_t + \gamma_{t+1}\delta\pi n_1 + \gamma_t n_1(1-\pi)}\right)\frac{u'(c_t^1)}{u'(d_{t+1}^1)} = \frac{\beta}{\delta}\frac{\lambda_t}{\theta_{t+1}}.$$

Joint with $\alpha = 1$ and (14g) we get

$$\left(\frac{n_1 - \mu_t + \gamma_t n_1(1-\pi)}{n_1 - \mu_t + \gamma_{t+1}\delta\pi n_1 + \gamma_t n_1(1-\pi)}\right)\frac{u'(c_t^1)}{u'(d_{t+1}^1)} = \beta(1+r) - (1-\pi)\frac{\gamma_{t+1}}{\theta_{t+1}}\beta u'(\bar{d}_t).$$

Again, a similar operation is conducted with type2 individuals (except that $n_1 - \mu_t$ is replaced by $n_2 + \mu_t$).