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Childlessness, Childfreeness and Compensation*

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Abstract

We study the design of a fair family policy in an economy where parenthood is regarded either as desirable or as undesirable, and where there is imperfect fertility control, leading to involuntary childlessness/parenthood. Using an equivalent consumption approach in the consumption-fertility space, we first show that the identification of the worst-off individuals is not robust to how the social evaluator fixes the reference fertility level. Adopting the ex post egalitarian social criterion, which gives priority to the worst off in realized terms, we then examine the compensation for involuntary childlessness/parenthood. Unlike real-world family policies, a fair family policy does not always involve positive family allowances to (voluntary) parents, and may also, under some reference fertility levels, involve positive childlessness allowances. Our results are robust to assuming asymmetric information and to introducing Assisted Reproductive Technologies.

Keywords: fertility, childlessness, family policy, compensation, fairness.

JEL codes: J13, I38.

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1 Introduction

Family policies are old, and date back, at least, to the Mercantilist epoch, that is, the first attempt to build a consistent national system of economic policies. For instance, there was, under the French Kingdom at the time of Colbert (17th century), a public pension offered to fathers of at least 12 children. The goal was to increase the number of births, at a time where a sizable population was regarded as a necessary condition for military and economic power.

More than three centuries after Colbert, it is quite surprising to see that, when economists and demographers evaluate the impact of family policies, they still focus mainly on their effect on fertility, that is, on the quantity of births per woman (see Gauthier 2007, Thévenon and Gauthier 2011). While family policies are multidimensional (family allowances, parental leave, day-care policies, etc.), most studies focused on the impact of family allowances on fertility, and often showed a positive effect of these allowances on the number of children.[1]

But family policies do not only have an impact on the number of births. These policies have also important distributive implications. Subsidizing births by means of uniform family allowances affects the distribution of income, to the extent that fertility behaviors vary with income. If children are inferior goods, subsidizing births redistributes resources towards lower income classes, whereas the opposite holds if children are superior goods.[2]

The goal of this paper is to reexamine family policies, and in particular family allowances, from the perspective of *fairness*. Instead of evaluating their impact on the size of the population (which is, in the light of the population ethics literature and its paradoxes, a quite questionable goal), we propose to consider family policies from a fairness perspective, that is, to regard family policies as instruments aimed at providing compensation to the disadvantaged.[3]

In the context of fertility, an important source of welfare disadvantage or deprivation consists of inequality in individual fecundity, i.e. the capacity to give birth to children. Humans are unequal in terms of their capacity to give birth to children. As shown by Leridon (1992) for France, about 10 % of each cohort remains childless because of purely biological reasons. That phenomenon is a case of *involuntary* childlessness. Involuntary childlessness is at the origin of large well-being inequalities within the population. Actually, since having children is regarded by many individuals as a fundamental component of their life-plans, remaining childless constitutes a major source of welfare deprivation.

That source of deprivation being (largely) exogenous, there is a strong case for compensating individuals who suffer from involuntary childlessness. Indeed, following the Principle of Compensation (Fleurbaey 2008, Fleurbaey and Mani-

[1] See Gauthier and Hatzius (1997), D'Addio and Mira d'Ercole (2005), Luci-Greulich and Thévenon (2014). One exception is Kalwij (2010), who finds that family allowances have no significant impact on fertility in Western Europe.

[2] On distributive effects, see Balestrino et al (2002) and Pestieau and Ponthiere (2013).

[3] On population ethics and its paradoxes, classical references are Parfit (1984) and Blackorby et al (2005). Those pieces of work question all standard social criteria in the context of varying population size, and, as such, question also a purely "productivist" evaluation of family policies in terms of their impact on the number of births.

quet 2010), a government should intervene to abolish well-being inequalities that are due to circumstances. Given that involuntary childlessness is mainly due to biological circumstances, there is here a strong ethical argument supporting the compensation of the involuntary childless.

Note, however, that the childlessness phenomenon includes not only involuntary childlessness, but, also, *voluntary* childlessness, that is, some men and women make the choice to remain without children. Actually, about 15 % of a women cohort remains childless, with significant variations across countries and epochs (see Sobotka 2017).[4] Given that involuntary childlessness due to biological causes amounts to about 10 % of a cohort, voluntary childlessness concerns about 5 % of a cohort.[5] When referring to women who decided not to have a child, the negatively-valued term "childlessness" is often replaced by the more positive term "childfreeness", i.e. being free from children.

Heterogeneity in preferences towards children raises deep challenges for the design of optimal family policies. Heterogeneous preferences complicate interpersonal well-being comparisons. The design of a fair family policy requires to make the well-being of the involuntary childless (who regards children as desirable) and of the childfree (who regards children as undesirable) *comparable*. Moreover, in real-world economies, it is difficult for governments to observe preferences, and to distinguish between the childless and the childfree. Asymmetric information makes compensation for childlessness even more challenging.

Another major challenge for the design of a fair family policy is the development, in the last decades, of Assisted Reproductive Technologies (ART) (see Trappe 2017, Prag and Mills 2017). Those new technologies can reduce the number of individuals to be compensated for the damage of involuntary childlessness. However, ART treatments are extremely costly, and, here again, asymmetric information can be challenging for the design of a fair family policy. Under scarce resources, governments should restrict access of these extremely costly techniques to individuals who (i) really want children and (ii) cannot have children otherwise, two features that are, once again, hard to observe.

The goal of this paper is to revisit the design of optimal family policy, by paying a particular attention to the compensation of the involuntary childlessness in an economy peopled of individuals for whom parenthood is regarded either as a desirable or as undesirable, and where there is imperfect contraceptive and reproductive technology, which can lead to involuntary parenthood/childlessness.

In order to examine the design of a fair family policy, we proceed in five stages. First, we develop a model of binary fertility (either 0 or 1 child), with imperfect fertility control and heterogeneity on preferences towards children. Second, we address the challenge of interpersonal well-being comparisons by using an equivalent consumption approach, which amounts to construct an inclusive, preferences-based, index of well-being that relies on reference achievements for

[4] The childlessness phenomenon is less widespread in France in comparison to Germany or the UK (Koppen et al 2017, Berrington 2017, Kreyenfeld and Konietzka 2017).

[5] This is in line with Toulemon (2001), who shows, in the case of France, that about 5 % of individuals state that childlessness is the most ideal living arrangement, whereas most men and women say that 2 or 3 is the ideal number of children. See also Kuhnt et al (2017).

non-monetary dimensions of well-being (Fleurbaey and Maniquet 2011, Fleurbaey and Blanchet 2013, Fleurbaey 2016). Third, in order to do justice to the idea of compensation, we adopt the ex post egalitarian social criterion, which consists of a maximin on consumption equivalents defined for a given reference fertility, and we characterize the ex post egalitarian social optimum. Fourth, assuming perfect observability of preferences, we study the decentralization of the ex post egalitarian optimum by means of a mixed adoption/transfer system. Fifth, we study the robustness of the fair family policy to introducing asymmetric information on preferences and ART investments.

Our analysis of a fair family policy leads us to two main results.

First, the design of the fair family policy is not robust to how the social evaluator fixes the reference fertility level, which is defined as the level of fertility at which interpersonal well-being comparisons can be made by merely comparing individual consumption levels. The reference fertility level, by determining who is the worst-off, definitely affects the ex post egalitarian optimum and the policies decentralizing that optimum. It also influences the conditions under which the ex post egalitarian optimum can be decentralized.

Second, our analysis shows that the fair family policy differs from existing family policies. The fair family policy does not always involve positive family allowances to voluntary parents, and may also, under some reference fertility, involve positive childlessness allowances, unlike real-world family policies. That result is robust to the introduction of asymmetric information on individual preferences (even though the allowance on involuntary childless parents is then reduced), and to the inclusion of ART treatments. Under ART treatments, the fair family policy involves additional allowances compensating individuals for the monetary and psychological costs of using ART as well as an additional allowance to ART users who did not succeed in becoming parents.[6]

As such, this paper casts original light on the optimal design of a fair family policy, and on how it differs from existing family policies, taking into account new societal realities such as the development of ART. Shifting the objective from raising the population size to compensating the disadvantaged affects the optimal family policy in a way that is robust to introducing asymmetric information.

This paper is related to several branches of the literature. First, it is related to the literature on childlessness. As far as demographers are concerned, Kreyenfeld and Konietzka (2017) synthesizes research on the determinants and dynamics of the childlessness phenomenon. On the economic side, Gobbi (2013) and Baudin et al (2015, 2019) study the determinants of childlessness in the U.S. and around the world. Childlessness is also studied by Etner et al (2016), who examine the impact of childlessness on long-run growth, while adopting a macroeconomic perspective (with same preferences). We complement those studies by considering childlessness from a normative perspective, to characterize a fair family policy under heterogeneous preferences. Second, our work is

[6] Again, those allowances remain - but are reduced - when asymmetric information is introduced.

also related to the public economics literature on optimal policy under varying fertility, such as Cigno (1983, 1986), Cremer et al (2006), Cremer et al (2008) and Pestieau and Ponthiere (2013). The specificity of our paper is to adopt a fairness perspective, and to consider family policy as an instrument aimed at compensating the involuntary childless and/or the involuntary parent. Finally, our work is also related to the welfare economics literature on compensation (Fleurbaey 2008, Fleurbaey and Maniquet 2010). That literature has, in the recent years, given rise to various applications, in particular concerning the compensation for unequal lifetimes (Fleurbaey et al 2014, Fleurbaey et al 2016). This paper complements that literature by considering the other end of the demographic chain, that is, compensation for unequal fecundity.

The paper is organized as follows. Section 2 presents a model of imperfect fertility control and heterogeneous preferences. The identification of the worst-off is carried out in Section 3 while adopting the equivalent consumption approach. The ex post egalitarian optimum is characterized in Section 4, which examines also its decentralization. Section 5 introduces asymmetric information on preferences. Section 6 considers ART treatments. Section 7 concludes.

2 The model

Let us consider an economy whose population of adults is a continuum of size 1. The adult population is composed of two categories of individuals: on the one hand, individuals who regard children as a *desirable* good (who are in a proportion $0 < x < 1$ in the population); on the other hand, individuals who regard children as an *undesirable* good (who are in a proportion $1 - x$).

Fertility is assumed to be binary and imperfectly controlled: individuals can either remain childless, or have one child, with a probability that depends on whether they consider children to be desirable or not (see below).

Preferences Individuals who consider children as desirable have additively-separable preferences in consumption and fertility, given by:

$$u(c) + v(n) \tag{1}$$

where c is consumption and $n = \{0, 1\}$ is the number of children. We assume that $u(\cdot)$ is increasing and concave. $v(\cdot)$ is increasing with $v(0) = 0$.

The preferences of individuals who consider children as undesirable are also additively-separable in consumption and fertility:

$$U(c) - V(n) \tag{2}$$

where $U(\cdot)$ is increasing and concave. $V(\cdot)$ is increasing and satisfies $V(0) = 0$.

We allow the functions $u(\cdot)$ and $U(\cdot)$ to differ. This modeling allows the marginal utility of consumption to differ between individuals differing in their taste for fertility, $U'(c) \gtreqless u'(c)$.[7] The functions $v(\cdot)$ and $V(\cdot)$ can also differ, so that our modeling allows also for having an asymmetry between, on the one hand, the well-being gain from parenthood for individuals who regard children as desirable, and, on the other hand, the well-being loss from parenthood for individuals who regard children as undesirable.

Fertility technology Individuals have an imperfect control on the number of children $n \in \{0, 1\}$. Our framework lies somewhere between the standard model of perfect fertility control (see Barro and Becker 1989) and a model of purely random fertility (no control at all).

Let us assume that individuals who consider children as desirable have a child with probability $0 < \pi < 1$, and are childless with probability $1 - \pi$. We suppose also that individuals who consider children as undesirable have a child with probability $0 < \varepsilon < 1$, and are childless with a probability $1 - \varepsilon$. We assume, without loss of generality, that the probability to have a child is larger among individuals who consider children as desirable than among individuals who consider children as undesirable, that is: $\pi > \varepsilon$.

The gap $\pi - \varepsilon$ depends on the efficiency of reproductive and contraceptive technologies. Perfect control occurs when $\pi = 1$ and $\varepsilon = 0$. In that case, there is no involuntary childlessness/parenthood, which are the object of this paper. We thus assume imperfect fertility control, i.e. $\pi < 1$ and $\varepsilon > 0$.

Note also that a model with pure random fertility would involve $\pi = \varepsilon$ (same probability to have a child for all individuals, independently from their willingness to have a child). Allowing for a gap between π and ε prevents us from having that extreme, quite unrealistic, random fertility model, and to consider instead a more realistic model of imperfect fertility control.

Following Barro and Becker (1989), we assume that having a child implies a monetary cost $g > 0$, as well as a time cost qw, where $w > 0$ is the hourly wage and $0 < q < 1$ is the fraction of time dedicated to raising the child.

The laissez-faire economy Once the fertility outcome has realized (i.e. ex post), the economy is composed of four types of individuals:[8]

- Type 1: individuals who want to have a child and have a child, in proportion $x\pi$
- Type 2: individuals who want to have a child and do not have a child, in proportion $x(1 - \pi)$
- Type 3: individuals who do not want a child and have a child, in proportion $(1 - x)\varepsilon$

[7] The desire for children may reinforce the marginal utility of consumption (reinforcement effect: $u'(c) > U'(c)$ for a given c), or, alternatively, it may weaken the marginal utility of consumption (redundancy effect: $u'(c) < U'(c)$ for a given c).

[8] Under the Law of Large Numbers, probabilities π and ε will also determine, together with the parameter x, the proportions of the different types of individuals in the population.

- Type 4: individuals who do not want a child and do not have a child, in proportion $(1-x)(1-\varepsilon)$.

Let us compare the well-being W^i of these different groups at the laissez-faire, i.e. in the absence of family policy:

$$\begin{aligned} W^1 &\equiv u(w(1-q)-g)+v(1) \\ W^2 &\equiv u(w) \\ W^3 &\equiv U(w(1-q)-g)-V(1) \\ W^4 &\equiv U(w) \end{aligned}$$

It is reasonable to assume that the involuntary childless (type 2) are worse off than the lucky parents (type 1), so that $W^1 > W^2$:

Assumption 1 *Individuals who want to have a child are better off with a child than without it:* $u(w(1-q)-g)+v(1) > u(w)$.

We also have, given the monotonicity of $U(\cdot)$ and $V(\cdot)$, that individuals who have a child but do not want to have a child (type 3) are worse-off than those who do not want a child and do not have one (type 4), i.e. $W^3 < W^4$, since

$$U(w(1-q)-g)-V(1) < U(w)$$

Finally, concerning the comparison of types 2 and 4, it is reasonable to assume that $W^4 > W^2$, that is:

Assumption 2 *Individuals who do not want a child and have no child are better off than those who want a child and have no child:* $U(c) > u(c)$.

The above expressions do not allow us to provide a *complete* ranking of individuals in terms of well-being (in particular whether $W_1 \gtreqless W_4$ or $W_2 \gtreqless W_3$). The reason is that individuals do not share the same preferences: while types 1 and 2 want to have a child, types 3 and 4 prefer not to have one. The next section examines well-being comparisons by using equivalent consumption indexes.

3 Identifying the worst-off at the laissez-faire

Prior to the design of a fair family policy, a preliminary step consists of identifying worst-off individuals. Section 2 showed that individuals who suffer from involuntary childlessness are worse off than individuals who succeed in becoming parents. Moreover, individuals who suffer from involuntary parenthood are also worse-off than childfree individuals. But what about the comparison of involuntary childless individuals (type 2) with involuntary parents (type 3)?

To answer that question, this section builds on recent advances in welfare economics and makes interpersonal well-being comparisons by means of the equivalent income/consumption indexes (Fleurbaey and Maniquet 2011, Fleurbaey and Blanchet 2013, Fleurbaey 2016). The construction of the equivalent income/consumption index consists in building, from individual preferences, an inclusive index of well-being, which takes into account not only the material part of well-being, but, also, all non-monetary dimensions of well-being. As we show below, those non-monetary dimensions of well-being are included by fixing reference levels for all these, and by deriving the hypothetical income/consumption level which, combined with reference levels for all non-monetary dimensions, would make individuals as well-off as they are with their current situation.

In that approach, reference levels for non-monetary dimensions of well-being are ethical parameters, which allow for the interpersonal comparison of well-being across individuals who have different preferences. Situations are generally hard to compare when individuals have different preferences, but when individuals enjoy reference levels, it is sufficient, in order to rank their well-being levels, to compare the levels of income/consumption that they enjoy (see Fleurbaey 2016).

In the present context, we are in the consumption-fertility space, and so there is one non-monetary dimension, fertility, for which a reference level is to be fixed. Four alternative approaches are possible here:

R1 The reference fertility level is fixed to 0 child for all individuals.

R2 The reference fertility level is fixed to 1 child for all individuals.

R3 The reference fertility level is fixed to 1 child for those who want children, and to 0 child for those who do not want children.

R4 The reference fertility level is fixed to 0 child for those who want children, and to 1 child for those who do not want children.

This section studies the sensitivity of the identification of the worst-off to the postulated reference fertility, by constructing equivalent consumption indexes for all individuals under reference fertility R1 to R4.

Construction of equivalent consumption under R1 When the reference fertility level is fixed to $\bar{n} = 0$, equivalent consumptions, $\hat{c}^i$, satisfy:

$$\begin{aligned}
u(\hat{c}^1) &= u(w(1-q-g)) + v(1) \iff \hat{c}^1 > w(1-q-g) \\
u(\hat{c}^2) &= u(w) \iff \hat{c}^2 = w \\
U(\hat{c}^3) &= U(w(1-q-g)) - V(1) \iff \hat{c}^3 < w(1-q) - g < w \\
U(\hat{c}^4) &= U(w) \iff \hat{c}^4 = w
\end{aligned}$$

where on the left-hand sides of the first equalities, $V(\bar{n}) = v(\bar{n}) = 0$ and the right-hand-sides give the utility obtained by each type at the laissez-faire. Hence, under that reference fertility level and using Assumption 1, we have:

$$\hat{c}^3 < \hat{c}^2 = \hat{c}^4 < \hat{c}^1$$

Under R1, the worst-off is type-3 (involuntary parent). Voluntary parents (type 1) are strictly better off than childfree (type 4) and childless (type 2) individuals.

Construction of equivalent consumption under R2 Let us now fix the reference fertility level to $\bar{n} = 1$. Equivalent consumptions $\tilde{c}^i$ satisfy:

$$\begin{aligned}
u\left(\tilde{c}^1\right) + v(1) &= u(w(1-q)-g) + v(1) \iff \tilde{c}^1 = w(1-q) - g \\
u\left(\tilde{c}^2\right) + v(1) &= u(w) \iff \tilde{c}^2 < w \\
U(\tilde{c}^3) - V(1) &= U(w(1-q)-g) - V(1) \iff \tilde{c}^3 = w(1-q) - g \\
U(\tilde{c}^4) - V(1) &= U(w) \iff \tilde{c}^4 > w
\end{aligned}$$

Under Assumption 1, we have $\tilde{c}^1 > \tilde{c}^2$. Obviously, the last two lines yield that $\tilde{c}^4 > \tilde{c}^3$ and together with $\tilde{c}^1 = \tilde{c}^3$, we obtain:

$$\tilde{c}^2 < \tilde{c}^1 = \tilde{c}^3 < \tilde{c}^4$$

Hence the worst-off is type-2 (involuntary childless). Childfree individuals (type 4) are here regarded as better off than voluntary (type 1) and involuntary (type 3) parents .

Construction of equivalent consumption under R3 A third approach consists in fixing reference to *levels that would be optimal* for individuals based on their preferences, that is, to $\bar{n}_{1,2} = 1$ for individuals who want a child (types 1 and 2), and to $\bar{n}_{3,4} = 0$ for individuals who do not want a child (types 3 and 4). Equivalent consumptions, $\bar{c}^i$ satisfy:

$$\begin{aligned}
u\left(\bar{c}^1\right) + v(1) &= u(w(1-q)-g) + v(1) \iff \bar{c}^1 = w(1-q) - g \\
u\left(\bar{c}^2\right) + v(1) &= u(w) \iff \bar{c}^2 < w \\
U(\bar{c}^3) &= U(w(1-q)-g) - V(1) \iff \bar{c}^3 < c^3 = w(1-q) - g \\
U(\bar{c}^4) &= U(w) \iff \bar{c}^4 = c^4 = w
\end{aligned}$$

Under Assumption 1, we have

$$\bar{c}^2, \bar{c}^3 < \bar{c}^1 < \bar{c}^4.$$

Note that, under that alternative reference fertility, it is no longer possible to identify the worst-off, since it depends on the specific forms of $u(\cdot), v(\cdot), U(\cdot), V(\cdot)$. However, given the existence of time costs and good costs of children, childfree individuals are better off than voluntary parents (like under R2).

Construction of equivalent consumption under R4 A fourth approach consists of fixing reference fertility to levels that individuals want to avoid based on their preferences, that is, to $\bar{n}_{1,2} = 0$ for individuals who want a child (types 1 and 2), and to $\bar{n}_{3,4} = 1$ for individuals who do not want a child (types 3 and 4). Equivalent consumptions $\breve{c}^i$ satisfy:

$$\begin{aligned}
u(\breve{c}^1) &= u(w(1-q)-g) + v(1) \iff \breve{c}^1 > w(1-q) - g \\
u(\breve{c}^2) &= u(w) \iff \breve{c}^2 = w \\
U(\breve{c}^3) - V(1) &= U(w(1-q)-g) - V(1) \iff \breve{c}^3 = w(1-q) - g \\
U(\breve{c}^4) - V(1) &= U(w) \iff \breve{c}^4 > w
\end{aligned}$$

Hence, under Assumption 1, we have that:

$$\breve{c}^3 < \breve{c}^2 < \breve{c}^1, \breve{c}^4$$

and that the worst-off is the involuntary parent (type 3).

All in all, the identification of the worst-off is not robust to the postulated reference fertility. Proposition 1 summarizes our results.

Proposition 1 *The identification of the worst-off individual is not robust to the postulated reference fertility level.*

reference fertility	*worst-off individuals*
R1 ($\bar{n} = 0$)	*involuntary parents*
R2 ($\bar{n} = 1$)	*involuntary childless*
R3 ($\bar{n}_{1,2} = 1$, $\bar{n}_{3,4} = 0$)	*involuntary childless/parents*
R4 ($\bar{n}_{1,2} = 0$, $\bar{n}_{3,4} = 1$)	*involuntary parents*

Proof. See above. ■

Proposition 1 states a negative result. Indeed, if the identification of the worst-off were robust to the selected reference fertility level, one could base well-being comparisons on any of those reference levels, without any risk of lack of robustness. But Proposition 1 states that such a robustness does not prevail and that the selected reference fertility matters for well-being comparisons.

The negative result of Proposition 1 would also be less problematic if there existed an obvious, an almost “natural" candidate for the reference fertility level. For instance, when considering issues of health, the good health status is a “natural" reference.[9] Similarly, when considering issues of life and death, the maximum lifespan is also a “natural" reference. But regarding fertility, such a “natural" candidate does not seem to exist.

The lack of robustness of the identification of the worst off to the postulated reference fertility has a major corollary for the design of a fair family policy. Given that the reference fertility affects how well-being levels are ranked, policy analysis should rely not on one, but on several reference fertility levels, to avoid the arbitrariness due to the selected reference. The next section characterizes the fair family policy under cases R1 to R4.

[9] See Fleurbaey (2005).

4 The ex post egalitarian optimum

In our model of imperfectly controlled fertility, individuals can hardly be regarded as responsible for (not) having a child. These are, in our model of imperfect fertility control, pure circumstances that determine fertility outcomes, and, hence, lead to the well-being inequalities studied above.

In situations where well-being inequalities are due to pure circumstances, the Principle of Compensation states that the government should intervene, so as to abolish those inequalities (see Fleurbaey, 2008, Fleurbaey and Maniquet, 2010). The underlying idea is that well-being inequalities due to circumstances are ethically unacceptable.

The Principle of Compensation thus requires to look for an allocation of resources that can lead to the compensation of individuals suffering from arbitrary well-being inequalities. In our model, this is the case for involuntary childless individuals, who cannot be regarded as responsible for their childlessness, as well as for involuntary parents, who cannot be regarded as responsible for their parenthood. A fair family policy should thus lead to equalize well-being levels between types 1 and 2, as well as between types 3 and 4.

As a preliminary step to the design of a fair family policy, this section characterizes the social optimum under the ex post egalitarian criterion, which aims at maximizing the *minimum* equivalent consumption, and at giving priority to the well-being of the worst-off ex post, i.e. once outcomes of fertility lotteries are known. As such, that social criterion does justice to the idea of compensating individuals for inequalities due to circumstances.[10]

4.1 The centralized solution

Let us first characterize the social optimum, by considering a benevolent planner who can allocate, within the population, not only material resources, but, also, children. This section characterizes the *optimum optimorum*, where the social planner can control all variables without any constraint except: (1) the total number of children; (2) the resource constraint.[11] In particular, the social planner can reallocate, at no cost, children, from individuals who are involuntary parents to individuals who are involuntary childless.[12]

Regarding the reallocation of children, three cases can arise, depending on the parameters (π, x, ε), which determine the relative size of the "demand for adopted children" (coming from type-2 individuals) with respect to the "supply for adopted children" (coming from type-3 individuals).[13]

[10] That social criterion was previously used by Fleurbaey et al (2014) in the context of unequal lifetimes.

[11] Surrogacy is not allowed here, so that the social planner takes the total number of children as given. The consequences of introducing surrogacy are studied in Section 4.4.

[12] For the sake of robustness to ethical foundations, Section 4.5 characterizes a constrained social optimum where the reallocation of children is prohibited.

[13] In each case, the reallocation of children is made randomly from one type to another.

Excess demand for adopted children That general case arises when $(1-\pi)\,x > \varepsilon\,(1-x)$. The reallocation of children from type-3 individuals to some type-2 individuals leads to an economy composed of voluntary parents of biological and adopted children (in proportion $x\pi + \varepsilon(1-x)$), involuntary childless individuals (in proportion $x(1-\pi) - \varepsilon(1-x)$) and childfree individuals (in proportion $1-x$). After reallocation of children, the social planner's problem consists of selecting consumptions $\left\{c^1, c^2, c^4\right\}$ so as to maximize the equivalent consumption of the worst-off, subject to the resource constraint:

$$\begin{array}{lcl} \max\limits_{c^1,c^2,c^4} \min & & \left\{C^1, C^2, C^4\right\} \\ & \text{s.t.} & \left[\begin{array}{l} (x\pi+\varepsilon(1-x))\,c_1 + (x(1-\pi)-\varepsilon(1-x))\,c_2 + (1-x)\,c_4 \\ +\,(x\pi+\varepsilon(1-x))\,g \end{array}\right] \\ & = & (x\pi+\varepsilon(1-x))\,w(1-q) + [(1-\pi)x + (1-x)]\,w \end{array}$$

where

$$C^i = \begin{cases} \hat{c}^i \text{ when } \bar{n} = 0 \text{ (R1)} \\ \tilde{c}^i \text{ when } \bar{n} = 1 \text{ (R2)} \\ \overline{c}^i \text{ when } \bar{n}_{1,2} = 1, \bar{n}_4 = 0 \text{ (R3)} \\ \breve{c}^i \text{ when } \bar{n}_{1,2} = 0, \bar{n}_4 = 1 \text{ (R4)} \end{cases}$$

Equality of demand and supply for adopted children That specific case arises when $(1-\pi)\,x = \varepsilon\,(1-x)$. All involuntary childless individuals are assigned a child, and all involuntary parents become childfree, so that the economy is left with only two categories, voluntary parents and childfree individuals. After reallocation of children, the social planner's problem consists of selecting consumptions $\left\{c^1, c^4\right\}$ so as to maximize the equivalent consumption of the worst-off, subject to the resource constraint:

$$\max_{c^1,c^4} \min \left\{C^1, C^4\right\} \text{ s.t. } xc_1 + (1-x)\,c_4 + xg = xw(1-q) + (1-x)w$$

where C^i are defined above.

Excess supply for adopted children That case arises when $(1-\pi)\,x < \varepsilon\,(1-x)$. After the reallocation of children, all involuntary childless become voluntary parents (of an adopted child), while only some involuntary parents become childfree. The economy is only composed of voluntary parents (in proportion x), involuntary parents (in proportion $(1-x)\varepsilon - x(1-\pi)$) and childfree individuals (in proportion $(1-x)(1-\varepsilon) + x(1-\pi)$). After the reallocation, the social planner selects consumptions $\left\{c^1, c^3, c^4\right\}$ so as to maximize the equivalent consumption of the worst-off, subject to the resource constraint:

$$\begin{array}{lcl} \max\limits_{c^1,c^3,c^4} \min & & \left\{C^1, C^3, C^4\right\} \\ & \text{s.t.} & \left[\begin{array}{l} xc_1 + ((1-x)\varepsilon - x(1-\pi))\,c_3 + ((1-x)(1-\varepsilon) + x(1-\pi))\,c_4 \\ +xg + ((1-x)\varepsilon - x(1-\pi))\,g \end{array}\right] \\ & = & (x + (1-x)\varepsilon - x(1-\pi))\,w(1-q) + ((1-x)(1-\varepsilon) + x(1-\pi))\,w. \end{array}$$

Those problems are solved in the Appendix. Proposition 2 summarizes our results.[14]

Proposition 2 *Assume that the economy is sufficiently productive. At the ex post egalitarian optimum, equivalent consumption levels are equalized for all, and we have:*

reference fertility	*excess demand*	*excess supply*	*equal demand and supply*
R1 ($\bar{n} = 0$)	$c^1 < c^2 = c^4$	$c^1 < c^4 < c^3$	$c^1 < c^4$
R2 ($\bar{n} = 1$)	$c^4 < c^1 < c^2$	$c^4 < c^1 = c^3$	$c^4 < c^1$
R3 ($\bar{n}_{1,2} = 1$, $\bar{n}_{3,4} = 0$)	$c^1 = c^4 < c^2$	$c^1 = c^4 < c^3$	$c^1 = c^4$
R4 ($\bar{n}_{1,2} = 0$, $\bar{n}_{3,4} = 1$)	$c^1, c^4 < c^2$	$c^1, c^4 < c^3$	$c^1 \gtreqless c^4$

Proof. See the Appendix. ■

Proposition 2 states that, provided the economy is sufficiently productive, the ex post egalitarian optimum involves a full equalization of equivalent consumption levels, leading to a full compensation for the childless and for the involuntary parents. The intuition behind that full compensation result comes from the fact that the postulated structure of individuals' preferences allows for some *substitutability* between consumption and parenthood. That substitutability allows to compensate involuntary childless individuals and involuntary parents by means of extra consumption.[15]

Another important contribution of Proposition 2 is to show that the ex post egalitarian optimum is sensitive to the postulated reference fertility level. To see this, take, for instance, the case where there is excess demand for adopted children. In that case, all remaining involuntary childless (type 2) should receive higher consumption than any other remaining type of individuals (1 and 4). However, the ranking of consumptions between voluntary parents and childfree individuals depends on the postulated reference fertility level.[16]

4.2 Decentralization of the optimum

In order to decentralize the social optimum, we first assume the existence of an adoption agency, which operates with a zero cost of children reallocation. We also assume that the welfare associated to adopting a child is exactly equal to that of having a child of his own, and that leaving a child for adoption does not generate any welfare loss.[17] Once the reassignment of children from involuntary

[14] By "sufficiently productive", we mean that the wage rate w is sufficiently large so as to allow for the perfect equalization of all consumption equivalents. This assumption is elicited in Section 4.3 for particular forms for $u(\cdot)$, $U(\cdot)$, $v(\cdot)$ and $V(\cdot)$.

[15] Alternatively, if there were perfect complementarity between consumption and parenthood, transfers would not allow to achieve full compensation.

[16] individuals, whereas the opposite holds under R2. Under R3, they enjoy equal consumptions, while under R4, the ranking between c_1 and c_4 depends on the forms of $u(\cdot)$, $U(\cdot)$, $v(\cdot)$ and $V(\cdot)$.

[17] Those assumptions are discussed below.

parents to involuntary childless has been made, we assume that the government can use monetary transfers to implement optimal allocations.

The timing of the decentralization is the following one:

Stage 1 Each parent of a child truthfully reveals whether his child was desired.

Stage 2 In case of a negative response, the parent sends his child to the adoption agency that takes care of all children available for adoption.

Stage 3 Each childless person truthfully reveals whether he wanted to have a child.

Stage 4 In case of an affirmative answer, the parent goes to the adoption agency and adopts a child provided a child is available for adoption.

Stage 5 After all possible adoptions have taken place, the government carries out monetary transfers aimed at equalizing equivalent consumption levels based on the postulated reference fertility.

In steps 1-4, the adoption agency reallocates children from involuntary parents (type 3) to the involuntary childless (type 2), which leads to Pareto-improvements in comparison to the laissez-faire.[18] Yet, introducing an adoption agency is not enough to achieve the first-best optimum, because of two reasons. First, in case of excess demand/supply for children, some individuals remain, after the reallocation, involuntary childless or involuntary parent. Second, even when demand equals supply, there is no equalization of consumption equivalents since parents and childfree have different resources resulting from raising a child or not. Thus a system of monetary transfers is also needed. Proposition 3 summarizes our results.

Proposition 3 *Assume that the economy is sufficiently productive. The decentralization of the ex post egalitarian optimum is achieved as follows:*

1. *If there is an excess demand for children, every child sent to adoption is adopted and all involuntary parent (type-3) become childfree (type-4). Transfers have the following form:*

reference fertility	*excess demand*
R1 ($\bar{n} = 0$)	$b_1 < 0 < b_2 = b_4$
R2 ($\bar{n} = 1$)	$b_4 < 0 < b_1 < b_2$ *or* $b_4 < b_1 < 0 < b_2$
R3 ($\bar{n}_{1,2} = 1$, $\bar{n}_4 = 0$)	$b_4 < 0 < b_1 < b_2$ *or* $b_4 < b_1 < 0 < b_2$
R4 ($\bar{n}_{1,2} = 0$, $\bar{n}_4 = 1$)	$b_4, b_1 < 0 < b_2$ *or* $b_1 < 0 < b_4 < b_2$ *or* $b_4 < 0 < b_1 < b_2$

2. *If there is equality between supply and demand of children, there remains only voluntary parents of a child (either adopted or not) and childfree individuals. Transfers have the following form:*

[18] Under assumption $A1$, the involuntary childless are better off adopting a child, and involuntary parents are better off leaving their child (since $U(w(1-q)-g) - V(1) < U(w)$).

reference fertility	*equal demand and supply*
R1 ($\bar{n} = 0$)	$b_1 < 0 < b_4$
R2 ($\bar{n} = 1$)	$b_4 < 0 < b_1$
R3 ($\bar{n}_1 = 1$, $\bar{n}_4 = 0$)	$b_4 < 0 < b_1$
R4 ($\bar{n}_1 = 0$, $\bar{n}_4 = 1$)	$b_1 < 0 < b_4$ *or* $b_4 < 0 < b_1$

3. *If there is an excess supply of children, not every child sent to adoption is adopted and no involuntary childless individual (type-2) remains. Transfers have the following form:*

reference fertility	*excess supply*
R1 ($\bar{n} = 0$)	$b_1 < 0 < b_4 < b_3$ *or* $b_1 < b_4 < 0 < b_3$
R2 ($\bar{n} = 1$)	$b_4 < 0 < b_1 = b_3$
R3 ($\bar{n}_1 = 1$, $\bar{n}_{3,4} = 0$)	$b_4 < b_1 < 0 < b_3$ *or* $b_4 < 0 < b_1 < b_3$
R4 ($\bar{n}_1 = 0$, $\bar{n}_{3,4} = 1$)	$b_4, b_1 < 0 < b_3$ *or* $b_1 < 0 < b_4 < b_3$ *or* $b_4 < 0 < b_1 < b_3$

Proof. See the Appendix. ■

An important result of Proposition 3 is the lack of robustness of the fair family policy to the reference fertility. Depending on the reference fertility, policies decentralizing the ex post egalitarian optimum can take various forms.

Another crucial result concerns the comparison of the fair family policy with real-world family policies. Departures are significant on two grounds.

First, Proposition 3 shows that it is far from being always the case that voluntary parents should obtain a positive child allowance (i.e. $b_1 > 0$). In fact, under R1, voluntary parents always have to pay a child tax, while under R2 to R4, they may receive a transfer or pay a tax. As such, Proposition 3 reveals that a fair family policy would take a quite different form from actual ones where children are in general subsidized.

The other interesting difference with respect to real-world family policies consists in the existence of allowances for *childless* individuals, which are always positive for the involuntary childless individuals and can be positive or negative for the voluntary childless individuals depending on the reference fertility level. This is, here again, a fundamental departure from actual policies, where only parents benefit from family policies.

This being said, one should also remind that our analysis makes some simplifying assumptions. First, it is assumed that the adoption system can be implemented at a zero cost. Yet, introducing a cost of adoption would not modify our results.[19] Second, we assume that involuntary parents leave their child to the adoption agency without any regret or guiltiness. Instead, one could add an extra psychological cost for leaving a child to the adoption agency. Again, this would only affect the shape of the monetary transfers to be made, provided such monetary compensation can be a substitute for the psychological cost. Finally, we assume that there is perfect substitutability between an adopted child and

[19]Indeed, in case of an individual cost, this would require to give additional monetary transfers to involuntary childless who would engage in the process of adopting a child. If it is a cost to the society as a whole, this would enter the government budget constraint and modify the amounts of lump-sum transfers to be made.

a biological child. Again, one could relax that assumption and provide different monetary compensations for parents of an adopted child or of a biological one.

4.3 The quasi-linear case

Section 4.2 assumed that the economy is "sufficiently productive", so that monetary resources needed to make all transfers are sufficiently large. In order to make explicit the meaning of "sufficiently productive economy", Proposition 4 identifies, for the case of quasi-linear preferences, conditions on the hourly wage w that are necessary and sufficient for the decentralization of the social optimum by means of a mixed adoption/monetary transfer system. For the sake of presentation, it focuses on the case of excess demand for children, which is the most realistic one, when reference fertility is fixed to 0 (R1) or 1 (R2).[20]

Proposition 4 *Assume quasi-linear preferences. Assume that there is excess demand for children, i.e.* $(1-x)\varepsilon < x(1-\pi)$. *Define*

$$\tilde{w}_1 \equiv \frac{g\left[x\pi + \varepsilon(1-x)\right] + \left[x(1-\pi) + (1-x)(1-\varepsilon)\right] v(1)}{1 - \varepsilon q(1-x) - qx\pi}$$

$$\tilde{w}_2 \equiv \frac{g\left(x\pi + \varepsilon(1-x)\right) - (1-x)V(1) + [x(1-\pi) - (1-x)\varepsilon]v(1)}{1 - \varepsilon q(1-x) - qx\pi}$$

- *Under R1 ($\bar{n} = 0$), the social optimum can be decentralized by a mixed system if and only if $w > \tilde{w}_1$.*
- *Under R2 ($\bar{n} = 1$), the social optimum can be decentralized by a mixed system if and only if $w > \tilde{w}_2$.*
- *Threshold hourly wage levels are such that $\tilde{w}_2 < \tilde{w}_1$.*

Proof. See the Appendix. ■

Proposition 4 states that the decentralization of the social optimum can only be achieved provided the hourly wage is above some threshold levels $\tilde{w}_1$ and $\tilde{w}_2$. These thresholds depend on the form of individual preferences, and, in particular, on the intensity of the taste for parenthood (i.e. $v(1)$ and $V(1)$). Hence, the capacity of the economy to compensate financially individuals for parenthood or for childlessness depends on how much these individual value being involuntary parent or being involuntary childless.

Also, what "sufficiently large" means depends on the postulated reference fertility level. Indeed, the threshold wage level required for decentralization of the optimum when $\bar{n}$ is fixed to 0 is *lower* than when $\bar{n}$ is fixed to 1. It is thus possible to think about an economy where the hourly wage lies between the two thresholds $\tilde{w}_2$ and $\tilde{w}_1$, with the corollary that this mixed adoption/transfer system could decentralize the social optimum when $\bar{n}$ is fixed to 1, but not when $\bar{n}$ is fixed to 0. As such, Proposition 4 shows the impact of reference fertility on the possibility of decentralizing the social optimum.

[20] Similar conditions could be derived for cases R3 and R4.

4.4 Ethical aspects (1): surrogacy

Up to now, we considered a social planning problem where the total number of children is taken *as given*. By doing so, we implicitly assumed that surrogacy is not an ethically feasible option. Surrogacy is a legal agreement by which an individual agrees to conceive a child on behalf of another person, who will be the parent of the child later on. This option was not possible in this section.

Note that the legal status of surrogacy varies around the world (from acceptance to prohibition). Discussing ethical arguments for or against surrogacy would lead us far beyond the scope of this paper. However, it is interesting to examine the robustness of our results to introducing surrogacy as an available instrument for the social planner.

Under an initial excess demand for children, institutionalizing surrogacy would allow the social planner, by asking to some type-1 individuals to serve as surrogates, to achieve the equalization of the demand and the supply for children. Whatever the size of the initial excess demand is, it is possible that some type-1 individuals have more than one child, and reallocate those children to the involuntary childless (type 2), so as to equalize demand and supply for children. In our model, there is no cost of conceiving a child, only a cost of raising the child.[21] Hence, introducing surrogacy would lead us to the second case of the centralized solution studied above, where the population is composed only of voluntary parents and childfree individuals.

Thus, our baseline model can be easily modified to introduce surrogacy.[22] Note also that, if one added some costs of conceiving a child, then the social planning problem would be modified, and the optimal policy would include transfers compensating the surrogates for the extra cost of conceiving a child. Proposition 5 summarizes our results.

Proposition 5 *Assume that surrogacy is an available instrument.*

- *Under an initial excess demand for children, the decentralization of the ex post egalitarian optimum is achieved by first asking some type-1 individuals to serve as surrogates for type-2 individuals. Then, once the equality of demand and supply for children is achieved, the fair family policy is the one in Proposition 3, bullet list item 2.*

Proof. See above. ■

Thus, under an initial excess demand for children, surrogacy can be part of a fair family policy, and be a substitute for childlessness allowances considered in Proposition 3. While that solution allows to equalize well-being levels at a higher level than in the absence of surrogacy, it should be reminded that allowing for surrogacy as an available instrument is itself an ethical position. We are thus not comparing a "first-best" with a "second-best" optimum, but two first-best optima characterized by different ethical standards.

[21] Indeed, when children are reallocated away from involuntary parents, it is, from the perspective of the budget constraint, *as if* those parents did not have a child.

[22] Obviously, surrogacy can only help provided there is an initial excess demand for children.

4.5 Ethical aspects (2): no child reallocation

Another ethical aspect is worth being considered here. This other aspect is not about relaxing an ethical constraint (like for surrogacy), but about adding an extra ethical constraint. Up to now, we assumed that no ethical constraint regulates the reallocation of children. If one focuses only on the point of view of adults, such a reallocation can be easily defended, on the grounds of the Pareto-improvements that it allows. However, things become more complex once one wants to take into account the point of view of children.

Take, for instance, the case where an involuntary parent would like to become childfree, whereas his child does not want to be adopted by another parent. Reallocating that child to an involuntary childless person would no longer be a Pareto-improvement, since this would make the child worse-off.

To avoid such difficulties, one may impose a restriction on the reallocation of children. Given that a reallocation of children may lead to a worsening of the situation of some children, who are the most vulnerable individuals, but whose interests are not easily observable, one may argue in favor of introducing an ethical constraint on the family policy, requiring that *only monetary transfers are allowed, but not transfers of children.*

That "no child transfer" restriction is quite conservative, but can be justified on two grounds. First, from a consequentialist perspective - but including children -, the social evaluator may want to avoid reallocations of children that could worsen the situation of those most vulnerable persons.[23] Second, one may depart from a consequentialist perspective, and regard the family policy as a process that has, in itself, an ethical value. From that perspective, the "no child transfer" condition could be a component of the fair process itself.

In the Appendix, we solve the social planner's problem under the "no child transfer" constraint, and compare the associated fair family policy with its form under the unconstrained case. Proposition 6 summarizes our results.

Proposition 6 *Assume the "no child transfer" condition.*

- *The (constrained) fair family policy depends on the reference fertility level, and involves, in general, two departures from real-world family policies: (1) voluntary parents may not obtain a positive allowance; (2) involuntary childless may receive a positive allowance.*

- *Under quasi-linear preferences, the wage threshold allowing for the equalization of all consumption equivalents is higher under the "no child transfer" system than under the mixed adoption / monetary transfer system. Hence, in some cases, the equalization of all consumption equivalents can be achieved by the mixed system, but not by the pure transfer system.*

Proof. See the Appendix. ■

[23] Given that the interests of children are not easily observable, prohibiting *all* reallocations of children is a way to avoid reallocations that could worsen the situation of *some* of them.

Proposition 6 shows that imposing the "no child transfer" constraint does not qualitatively affect most of our results. It remains true that the form of the (now constrained) ex post egalitarian optimum - as well as the fair family policy - depends on the reference fertility (cases R1 to R4).

However, an interesting difference is that, once the family policy involves only pure transfers, it becomes more difficult to equalize all consumption equivalents. Under the adoption/ transfer system, children's reallocation allowed to achieve Pareto-improvements among adults at zero cost, which contributed to save resources available for compensating the remaining involuntary childless/parents. The "no child transfer" condition prevents this. Hence, it may be the case that the pure transfer system *cannot* equalize consumption equivalents, whereas the adoption/ transfer system can.

5 Asymmetric information on preferences

Up to now, we assumed that the government can observe individual preferences, and, hence, can distinguish between voluntary and involuntary parents and between voluntary and involuntary childless individuals. This section examines how the fair family policies would be modified under the more realistic assumption of asymmetric information on individual preferences.[24]

5.1 Incentive-compatibility under the mixed system

Let us first study whether the allocations presented in Proposition 2 are incentive-compatible when the government cannot observe preferences.

When thinking about the incentive-compatibility of the mixed adoption/transfer system, it is tempting, at first glance, to believe that the mixed system satisfies incentive-compatibility constraints, since the adoption system (stages 1 to 4), *when taken separately*, would make individuals reveal their true type, and either obtain a child when they want one (type 2), or send him to adoption when they do not want one (type 3).[25] However, the problem is more complex: if the structure of the mixed system is common knowledge, individuals anticipate, when announcing their types in early stages, that they may potentially benefit from more advantageous transfers in stage 5 by lying on their type.

To see that, let us first consider the case of excess demand.[26] Looking at the first-best allocation where $c_2 \geq c_4$ for any reference fertility level, childfree individuals (type 4) may have an incentive to pretend that they are involuntary childless (type 2). Indeed, if they do so, there is, under the mixed system, a probability $\left(1 - \frac{\varepsilon(1-x)}{(1-\pi)x}\right)$ that they will not be assigned a child, and benefit from compensatory transfers *as if* they were an involuntary childless person. Hence

[24] Quite realistically, we assume that having a child or not is observable, so that mimicking can never happen on this dimension.

[25] In other words, a pure adoption system (stages 1 to 4) would be incentive-compatible.

[26] When the demand for children equals the supply of children, the first-best allocation is incentive-compatible since only voluntary parents and childfree individuals remain.

the incentive-compatibility constraint takes the following form:

$$U\left(c_4\right) \geq \frac{\varepsilon\left(1-x\right)}{\left(1-\pi\right)x}\left[U(c_1)-V(1)\right]+\left(1-\frac{\varepsilon\left(1-x\right)}{\left(1-\pi\right)x}\right)U(c_2) \tag{3}$$

where on the LHS is the utility of a childfree when declaring his true type, and the RHS is his expected utility when pretending to be involuntary childless.

Under R1, the first-best allocation satisfies the above incentive constraint. However, under R2 to R4, nothing guarantees incentive-compatibility of first-best allocations. Depending on the forms of $U(\cdot)$ and $V(\cdot)$, and on the probability $\frac{\varepsilon(1-x)}{(1-\pi)x}$ to be assigned a child, it may be welfare-improving, in expected terms, for a childfree individual to pretend to be involuntary childless. For instance, if there is a strong rationing of children available for adoption, so that $\frac{\varepsilon(1-x)}{(1-\pi)x} \to 0$, the incentive-compatibility constraint simplifies to $U(c_4) \geq U(c_2)$. Except under R1, the first-best allocations (where $c_4 < c_2$) violate this constraint and this would be the case as long as $\frac{\varepsilon(1-x)}{(1-\pi)x}$ is low. But if the excess demand is small, i.e. $\frac{\varepsilon(1-x)}{(1-\pi)x} \to 1$, the incentive compatibility constraint writes $U(c_4) \geq U(c_1) - V(1)$, so that first-best allocations are implementable under R1 to R3, while this may not be the case in R4.[27]

Let us now consider the case of excess supply. In stage 5, only voluntary parents (type 1) may be tempted to pretend to be involuntary parents (type 3). Indeed, when they lie on their type, there is a probability $1-\frac{(1-\pi)x}{\varepsilon(1-x)}$ that they can keep their child, and benefit from type-3 compensatory transfers. Hence the incentive-compatibility constraint is:

$$u(c_1)+v(1) \geq \frac{\left(1-\pi\right)x}{\varepsilon\left(1-x\right)}u(c_4)+\left(1-\frac{\left(1-\pi\right)x}{\varepsilon\left(1-x\right)}\right)\left[u(c_3)+v(1)\right]. \tag{4}$$

Except under R2 (where the first-best allocation always satisfies the above incentive constraint), nothing guarantees that the first-best allocations are incentive-compatible. To see this, assume first that the excess supply of children is sufficiently large (i.e. $\frac{(1-\pi)x}{\varepsilon(1-x)} \to 0$) so that the probability to keep his child and to benefit from compensatory transfers is high. The above incentive constraint simplifies to $u(c_1) \geq u(c_3)$. The first-best allocations violate this constraint (except under R2) and this would be the case as long as $\frac{(1-\pi)x}{\varepsilon(1-x)}$ is low. On the contrary, when $\frac{(1-\pi)x}{\varepsilon(1-x)} \to 1$, the incentive-compatibility constraint becomes $u(c_1)+v(1) \geq u(c_4)$. Under R1 to R3, incentive constraints are satisfied.[28] But, depending on the form of $u(\cdot)$ and $v(\cdot)$, this may not be true under R4. Proposition 7 summarizes our results.

Proposition 7 *Assume asymmetric information on preferences,*

[27] Under R1 and R3, it is straightforward, since $c_1 \leq c_4$, while under R2, the first-best involves $\hat{c}_1 = c_1$ and $U(\hat{c}_4) - V(1) = U(c_4)$, so that the incentive-compatibility constraint is $U(\hat{c}_4) \geq U(\hat{c}_1)$. Since, in the first-best, we have $\hat{c}_4 = \hat{c}_1$, this constraint is binding.

[28] This is straigthforward under R2 and R3, since $c_1 \geq c_4$. Under R1, one needs to acknowledge that in the first-best, $\hat{c}_1 = \hat{c}_4$ and that $\hat{c}_1 = c_1$, $\hat{c}_4 = c_4$.

- *Under excess demand, if the probability to adopt a child is sufficiently low, first-best allocations under R2, R3 and R4 are not incentive-compatible.*

- *Under excess supply, if the probability to have his child sent to adoption is sufficiently low, first-best allocations under R1, R3 and R4 are not incentive-compatible.*

Proof. See above. ■

Proposition 7 states is that, in general, the first-best allocations are not implementable under asymmetric information. This justifies the design of second-best allocations that would take into account incentive-compatibility constraints.

5.2 Second-best allocation under the mixed system

Under excess demand, as shown in the previous section, only under R2 to R4, type-4 individuals may be tempted to declare to be type-2 individuals. Hence, the incentive compatibility constraint (eq. 3) needs to be satisfied. This can be done by increasing c_4 and decreasing c_1 and c_2 with respect to the first-best optimum, so that the incentive-constraint is binding. Depending on the forms of individual preferences and on the level of the probability that a type-4 would obtain a child, two rankings are now possible: either $c_4 < c_1 < c_2$ or $c_1 < c_4 < c_2$.[29] This, however, prevents the equalization of all consumption equivalents and full compensation. Indeed, in order to prevent mimicking from the childfree, one needs to leave them a rent, while decreasing the well-being of the involuntary childless and voluntary parents. As a result, at the second-best optimum, under R2, one now has $\tilde{c}_1$, $\tilde{c}_2 < \tilde{c}_4$ and $\tilde{c}_1 \gtrless \tilde{c}_2$.[30]

The same reasoning can be applied under excess supply under R1, R3 and R4. The second-best optimum has to prevent mimicking from type-1 individuals, so that the incentive-compatibility constraint (eq. 4) has to be satisfied. This can be done by increasing c_1 and decreasing c_3 and c_4, so as to ensure that this constraint is binding. This yields two possible ranking: either $c_4 < c_1 < c_3$ or $c_1 < c_4 < c_3$.[31] Again, in the second-best, full compensation cannot be achieved since, in order to avoid mimicking, type-1 individuals have to be left a rent and well-being of type-3 and type-4 must decrease. As a result, under R1, one has $\hat{c}_3$, $\hat{c}_4 < \hat{c}_1$ and $\hat{c}_3 \gtrless \hat{c}_4$.[32]

In sum, second-best allocations can be decentralized by using a mixed adoption/transfers system, but full equalization of all consumption equivalents cannot be achieved at the second-best, since informational rents are left either to the childfree or to voluntary parents so as to avoid mimicking.

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[29] Under R3, since in the first-best, $c_1 = c_4$, $c_1 < c_4 < c_2$ is the unique second-best solution.
[30] The same rankings of consumption equivalents are obtained for R3 and R4.
[31] In case R3, since in the first best, $c_1 = c_4$, $c_4 < c_1 < c_3$ is the unique second-best solution.
[32] The same rankings of consumption equivalents are obtained for R3 and R4.

6 Assisted reproductive technologies (ART)

Up to now, our analysis of a fair family policy assumed that the probability of becoming parent was *given*. One may want to relax that hypothesis, to allow individuals who want children to invest in assisted reproductive technologies (ART).[33] Such technologies, which are costly in terms of money and psychological strains, can increase the probability to have a child, and, as such, constitute a costly way to interfere with Nature's lottery.

6.1 The modeling of ART

The decision of investing in ART concerns only the involuntary childless (type-2), since type-1 individuals already have a child, while other types do not want one. For the sake of simplicity, let us assume that there exist only two levels of investment in ART: either $e = 0$ (no investment) or $e = \ell$ (full investment).[34] In addition to the pure monetary cost of ART, there is a psychological cost of investing in ART, $\varphi(\ell)$, with $\varphi(\ell) > \varphi(0) = 0$.

In the absence of any government intervention, type-2 individuals decide to invest in ART, $e = \ell$, if and only if

$$p[u(w(1-q) - g - \ell) + v(1) - \varphi(\ell)] + (1-p)[u(w-\ell) - \varphi(\ell)] \geq u(w) \quad (5)$$

where $0 < p < 1$ is the probability of having a child when investing in ART. The first part of the LHS is the utility of a type-2 individual who invests in ART and is successful, while the second part is the utility of that individual if he is unsuccessful. The RHS of the inequality is the utility of a type-2 individual who does not invest in ART and has no child. Since $u(w) > u(w-\ell) - \varphi(\ell)$, that inequality is satisfied if and only if:

$$u(w(1-q) - g - \ell) + v(1) - \varphi(\ell) > u(w) > u(w-\ell) - \varphi(\ell). \quad (6)$$

In the following, we assume that conditions (5) and (6) are always satisfied so that every involuntary childless (type-2) agent chooses to invest in ART.

Nonetheless, introducing ART does not imply that all involuntary childless agents disappear: some of those who invested in ART may turn out to be *unsuccessful* in having children. It simply complicates the analysis by first dividing the type of individuals who voluntarily have children in 2 subtypes: on the one hand, individuals who have a child naturally, and, on the other hand, individuals who invested in ART and were successful. Second, introducing ART also divides the involuntary childless type in 2 subtypes: on the one hand, childless individuals who invested in ART but were not successful, and, on the other hand, childless individuals who did not invest in ART.

For the sake of simplicity, we will abstract abstract from involuntary parents (previous type 3).[35] This leads us to 4 ex-post types:[36]

[33] On ART, see Trappe (2017). This section abstracts from surrogacy (see Section 4.4).

[34] Assuming, on the contrary, a continuum of values for the choice of ART investment would only complicate the model without adding any additional results.

[35] This amounts to assume that $\varepsilon = 0$ (i.e. perfect contraception).

[36] We use, here again, the Law of Large Numbers.

- Type 1: individuals who want to have a child and have a child with no investment in ART, in proportion $x\pi$.
- Type 2: individuals who want to have a child, cannot have a child, invest in ART and are successful, in proportion $x(1-\pi)p$.
- Type 3: individuals who want to have a child, cannot have a child, invest in ART and are unsuccessful, in proportion $x(1-\pi)(1-p)$.
- Type 4: individuals who do not want a child and do not have a child, in proportion $1-x$.

6.2 The laissez-faire economy

At the laissez-faire, ex post well-being levels of type-i individuals are:

$$\begin{aligned}
W^1 &\equiv u(w(1-q)-g)+v(1)\\
W^2 &\equiv u(w(1-q)-g-\ell)+v(1)-\varphi(\ell)\\
W^3 &\equiv u(w-\ell)-\varphi(\ell)\\
W^4 &\equiv U(w)
\end{aligned}$$

Since $\ell > 0$ and $\varphi(\ell) > 0$, we have, for individuals who want to have children, that $W^1 > W^2$. Under condition (6) for a positive investment in ART, it is also the case that $W^2 > W^3$. Moreover, Assumption A2 implies that $U(w) > u(w-\ell)-\varphi(\ell)$, so that $W^4 > W^3$: childfree individuals are better off than involuntary childless individuals.

Individuals who invested in ART but were unsuccessful (type 3) are unambiguously the worst-off. Having stressed this, one may suspect that the precise form of the ex post egalitarian optimum, and, hence, of the fair family policy, depends on reference levels for fertility $\bar{n}$ and for ART investment $\bar{e}$. Given that reference fertility only affects welfare comparisons among individuals with different preferences, these only affect the ranking of type 4 with respect to the other types (who all share the same preferences). Hence, we will focus only on the following reference levels for fertility and ART:

- R1 The reference fertility level is fixed to 0 child for all and the reference ART effort is fixed to 0 for all.
- R2 The reference fertility level is fixed to 1 child for all and the reference ART effort is fixed to 0 for all.
- R3 The reference fertility level is fixed to 1 child for all and the reference ART effort is fixed to ℓ for types 1-2-3 and 0 for type 4.

6.3 The ex post egalitarian optimum

The laissez-faire involves well-being inequalities, despite the introduction of ART. Those inequalities are due to circumstances that lie beyond the control of individuals. Therefore, the Principle of Compensation applies also to that alternative setting. It is thus legitimate to consider the question of the compensation of individuals who are childless despite the use of ART, as well as the compensation of individuals who could become parent, but at the cost of ART investments that other parents did not have to pay. We will thus, here again, rely on the ex post egalitarian social criterion, which does justice to the idea of compensating individuals for the damages due to circumstances.

A first interesting feature of the ex post egalitarian social optimum, is that, if the social planner were to choose both consumptions and ART investment for all individuals, the optimum would involve a zero level of ART investment. The intuition goes as follows. The worst-off is the one who invested in ART, but turned out to be unsuccessful in having children (type 3). From the perspective of maximizing the realized well-being of that individual, it would have been better not to invest in ART, since it involved costs, but did not allow him to have a child. Thus, if the social objective is to maximize the realized well-being of the worst-off, it is optimal to have a zero investment in ART.[37]

To examine the impact of introducing ART on optimal policies, we will assume that the government allows individuals to invest in ART, and selects the allocation of resources that maximizes the realized well-being of the worst off under that constraint, while leaving the allocation of children *unchanged*.[38] Note that, in theory, the government could, for the sake of improving the situation of the worst-off, reallocate children ex post, from type 1 (who could get a child easily) to type 3 (unsuccessful ART users). But those reallocations are not as ethically attractive as the ones in Section 4.1 (where children were taken away from *involuntary* parents, unlike here). There is then a strong case for applying the “no child transfer" condition here. Under that condition, the problem of social planner amounts to choose $\left\{c^1, c^2, c^3, c^4\right\}$ so as to maximize the equivalent consumption of the worst-off, subject to the resource constraint:

$$\begin{aligned}\max_{c^1,c^2,c^3,c^4} \quad & \min \quad \left\{C^1, C^2, C^3, C^4\right\} \\ \text{s.t.} \quad & \pi x c_1 + x(1-\pi)pc_2 + x(1-\pi)(1-p)c_3 + (1-x)c_4 + \pi x g + x(1-\pi)pg \\ = \quad & [\pi x + x(1-\pi)p]w(1-q) + x(1-\pi)(1-p)w + (1-x)w + x(1-\pi)\ell\end{aligned}$$

where

$$C^i = \begin{cases} \hat{c}^i \text{ when } \bar{n} = 0 \text{ and } \bar{e} = 0 \text{ (R1)} \\ \tilde{c}^i \text{ when } \bar{n} = 1,\, \bar{e}_{1,2,3} = 0 \text{ and } \bar{e}_4 = 0 \text{ (R2)} \\ \bar{c}^i \text{ when } \bar{n} = 1,\, \bar{e}_{1,2,3} = \ell \text{ and } \bar{e}_4 = 0 \text{ (R3)} \end{cases}$$

This leads to the following proposition:

[37] That result, which is close to the result of zero prevention (against mortality) in Fleurbaey and Ponthiere (2013), is due to the fact that there is a conflict between the goal of ex post compensation and the goal of investing in costly preventive processes that are successful with a probability less than unity.

[38] Note that, if we fixed ART to zero, we would be left with three types: individuals who want children and were successful in having children; individuals who want children and were not successful in having children, and childfree individuals. That framework would be a reduced form of the model studied in previous sections.

Proposition 8 *At the ex post egalitarian optimum with ART investments, equivalent consumption levels are equalized for the four types, and we have:*

reference fertility and ART	*consumption ranking*
R1 ($\bar{n} = 0$ *and* $\bar{e} = 0$)	$c^1 < c^2 < c^4 < c^3$
R2 ($\bar{n} = 1$ *and* $\bar{e} = 0$)	$c^4 < c^1 < c^2 < c^3$
R3 ($\bar{n} = 1, \bar{e}_{1,2,3} = \ell$ *and* $\bar{e}_4 = 0$)	$c^1, c^4 < c^2 < c^3$

Proof. See the Appendix. ■

Proposition 8 shows that, independently of the reference fertility and ART levels, the ex post egalitarian optimum always involves a higher consumption for type 3 (individuals who were unsuccessful despite ART) than for type 2 (individuals who were successful in using ART), and, also than for type 1 (parents who did not have to use ART). However, the ranking of the consumption of a type 4 (childfree individuals) with respect to other consumptions changes depending on the reference fertility and ART levels considered.

6.4 Decentralization through transfers

The decentralization of the ex post egalitarian optimum can be achieved by means of pure system of transfers. Proposition 9 summarizes our results.

Proposition 9 *Assume that the economy is sufficiently productive. The decentralization of the ex post egalitarian optimum with ART can be achieved by means of the following instruments:*

reference fertility/ART	*instruments*
R1 ($\bar{n} = 0$ *and* $\bar{e} = 0$)	*A positive allowance to unsuccessful ART users,* *a (smaller) positive/negative child allowance to successful ART users,* *a positive allowance to the childfree,* *a tax on parents who conceived naturally.*
R2 ($\bar{n} = 1$ *and* $\bar{e} = 0$)	*A positive allowance to unsuccessful ART users,* *a (smaller) positive/negative child allowance to successful ART users,* *and to parents who conceived naturally,* *a tax on the childfree.*
R3 ($\bar{n} = 1,\ \bar{e}_{1,2,3} = \ell,\ \bar{e}_4 = 0$)	*A positive allowance to unsuccessful ART users,* *a (smaller) positive/negative allowance to successful ART users,* *to parents who conceived naturally and to the childfree.*

Proof. See the Appendix. ■

Proposition 9 points to a clear policy recommendation. Whatever the reference for fertility and ART, the decentralization of the ex post egalitarian optimum requires to implement a positive allowance to ART users that would cover both monetary and psychological costs associated to ART. Indeed, unsuccessful ART users always obtain a positive allowance, while those who turned out to be successful receive a (smaller) allowance that may be positive or negative depending on the government budget constraint requirements as well as on preferences. Those features contrast with actual policies, where ART is generally not (fully) covered by public allowances, and where there is no additional compensation scheme for unlucky ART users.

Besides those common features, Proposition 9 also shows that the treatment of childfree individuals and of voluntary parents who conceived naturally varies with the reference fertility and ART levels. That result confirms what was already shown in the model without ART.

6.5 Second-best allocation

Let us now introduce asymmetric information on preferences, and suppose that the government only observes who has a child and who uses ART.

Under asymmetric information, whatever the reference fertility and ART (R1 to R3), if the social planner were to propose the first-best allocations (see Proposition 8), type-1 individuals, who have a child without using ART, may be tempted to use ART and pretend to be type-2 individuals so as to obtain higher consumption. In the same way, childfree individuals (type 4) may have an interest in investing in ART and pretend to be involuntary childless (type 3).[39] Hence, under asymmetric information, we need to add to the social planning problem two incentive-compatibility constraints:[40]

$$\begin{aligned} u(c_1) + v(1) &\geq u(c_2) + v(1) - \varphi(\ell) && (7) \\ U(c_4) &\geq U(c_3) - \varphi(\ell) && (8) \end{aligned}$$

where the LHS of the above inequalities is the utility of type-1 and type-4 individuals declaring honestly their type and the RHS is the utility they would obtain claiming to be a type 2 or 3 respectively.[41]

At the first-best allocation, for all reference levels (R1 to R3), the first incentive-compatibility constraint (7) is always binding, as a direct consequence of the equalization of the consumption equivalents of type-1 and type-2 individuals. Hence, a type-1 individual never has an interest in pretending to be a type-2. The intuition behind this is that when ART costs are fully observable, they make type-1 individuals obtain exactly the same utility as type-2 individuals since they obtain the same transfers, incur the full cost (monetary and non monetary) of ART and have the same preferences. To the opposite, the first-best

[39] These situations are possible if the costs of ART are not prohibitive.

[40] For simplicity, we assume here that childfree individuals have the same disutility of the ART treatment as other individuals.

[41] One could oppose that, instead of (7), the relevant incentive-compatibility constraint is:

$$u(c_1) + v(1) \geq p[u(c_2) + v(1) - \varphi(\ell)] + (1-p)[u(c_3) - \varphi(\ell)]$$

where the RHS is the *expected utility* of investing in ART. Assuming that a type-1 (who can conceive naturally with no ART help) has a probability $p \to 1$ when using ART, the two formulations are equivalent. The same reasoning can be applied in (8) for agents who do not want a child, by assuming that for them that the probability to have a child is $p \to 0$.

allocation may not always satisfy the second incentive constraint, so that type-4 individuals may be tempted to declare to be of type 3 if the first-best allocation were to be proposed. Here, the difference comes from the fact that, since type 3 and type 4 do not share the same preferences, the incentive constraint is not equivalent to comparing consumption equivalents.

Therefore, the second-best allocation should now satisfy the incentive constraint (8). For this to be the case, one needs to increase c_4 and decrease c_3 with respect to the first-best levels. Hence, consumption equivalents are not equalized anymore. For instance, under R1, we have now $\hat{c}_4 > \hat{c}_1 = \hat{c}_2 > \hat{c}_3$.[42] In that situation, childfree individuals (the potential mimickers) would obtain a rent and be left with higher consumption equivalents than any other type and unsuccessful ART users (type-3) would be left with less utility than the other categories, so as to avoid mimicking from the childfree.

Note, however, that the above second-best problem relies on the assumption that the government can observe the non-monetary cost of ART, so that all individuals who receive ART must face the disutility cost $\varphi(\ell)$. If one assumes, on the contrary, that only the monetary cost of ART is observable, but not its psychological cost, then individuals pretending to be needing ART could buy the treatment and throw it away, therefore not incurring the non-monetary cost of ART. In that case, the relevant incentive-compatibility constraints would be:[43]

$$
\begin{aligned}
u(c_1) + v(1) &\geq u(c_2) + v(1) &\quad (9)\\
U(c_4) &\geq U(c_3) &\quad (10)
\end{aligned}
$$

In that situation, the first-best allocations do not satisfy both incentive constraints, since in the first-best, $c_2 > c_1$ and $c_3 > c_4$ for any reference fertility and ART levels. Hence for the allocations to be incentive compatible, in the second-best, one should increase c_1 and decrease c_2 so as to get $c_1 = c_2$ and, increase c_4 and decrease c_3 so as to obtain $c_4 = c_3$. This goes against the idea of compensating ART users, that is, type 2 and type 3. In that situation, compensation and incentive compatibility cannot be achieved at the same time and, consumption equivalents are not equalized anymore: $\hat{c}_1 > \hat{c}_2$, $\hat{c}_4 > \hat{c}_3$ under R1, $\tilde{c}_1 > \tilde{c}_2$, $\tilde{c}_4 > \tilde{c}_3$ under R2 and $\bar{c}_1 > \bar{c}_2$, $\bar{c}_4 > \bar{c}_3$ under R3.

Proposition 10 *Let us assume that preferences are not observable to the government, while having a child and the purchase of ART are.*

- *If the non-monetary cost of ART is observable to the government, the first-best allocation is not implementable. The second-best allocation requires to increase consumption of the childfree and to decrease consumption of unsuccessful ART users as compared to the first-best levels.*

[42] We obtain the same rankings for the consumption equivalents in cases R2 and R3.

[43] One could oppose that the incentive-compatibility constraint has to include the *expected utility* of investing in ART. If we assume that type-1 (who can conceive naturally without ART) has a probability to have child, $p \to 1$ when using ART, and that agents who do not want a child have $p \to 0$, the two formulations are equivalent.

- *If the non-monetary cost of ART is not observable to the government, the first-best allocation is not incentive compatible and, consumption should be equalized between parents (whether they used ART or not) and between childless (whether they used ART or not).*

Proof. See above. ■

Whether the non-monetary cost of ART is observable or not, the first-best allocations are not implementable under asymmetric information. Yet, it appears that if the non-monetary cost is not observable, both incentive constraints are not satisfied, distorting the second-best allocations further. This gives an argument in favor of monitoring ART. Indeed, for a low cost of monitoring, the observability of non monetary ART costs enables to achieve more compensation than if it is not observable.

7 Conclusion

This paper proposes to cast a new light on family policies, by considering, instead of their capacity to promote fertility, their capacity to serve social justice in an economy where individuals are unequal in terms of fecundity. Our results show that shifting the goal from producing more children to achieving fairness has a strong impact on the design of family policies.

Our analysis first pointed out to a major difficulty when designing a fair family policy: the treatment of heterogeneity in preferences. Whereas some individuals think that having a child constitutes a fundamental dimension of their life-goals, other individuals prefer to avoid having children. In a world of imperfect fertility control, this heterogeneity leads to potentially two distinct types of damages: involuntary childlessness and involuntary parenthood. Our analysis based on the construction of equivalent consumption indexes revealed that the identification of the worst-off depends on the postulated reference fertility level, which plays a key role in interpersonal well-being comparisons.

We also showed that a fair family policy would differ strongly from family policies existing around the world. A fair family policy does not, in general, involve positive family allowances to voluntary parents, and may also, under some reference fertility, involve positive childlessness allowances, unlike real-world family policies. Note that, if one departs from that model and introduces surrogacy, it appears that institutionalizing surrogacy would make childlessness allowances unnecessary (since involuntary childlessness would not exist any more). If, instead, one allows for ART but not for surrogacy, a fair family policy involves an allowance compensating all ART costs, as well as an additional allowance for ART users that were unsuccessful in having children. Those results were shown to be robust to the introduction of asymmetric information on preferences, even though incentive-compatibility constraints lead to partial - instead of full - compensation for involuntary childlessness/parenthood.

In sum, considering family policies as instruments towards fairness has major implications for the design of those policies. Moreover, the form of the fair family policy is also sensitive to ethical judgements about available instruments for family policy, such as children's reallocation, ART and surrogacy.

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9 Appendix

9.1 Proof of Proposition 2

Excess demand of children Under R1, and once type 3 has disappeared thanks to the reallocation of children, the worst-off individuals are types 2 and 4 (see Section 3). Hence, the problem of the planner is:

$$\begin{array}{rl}\max\limits_{c^1,c^2,c^4} & \hat{c}^2 \\ & \text{s.t. } (\pi x+\varepsilon(1-x))c_1+(x(1-\pi)-\varepsilon(1-x))c_2+(1-x)c_4+(\pi x+\varepsilon(1-x))g \\ = & (\pi x+\varepsilon(1-x))w(1-q)+(x(1-\pi)+(1-\varepsilon)(1-x))w \\ & \text{s.t. } \hat{c}^1 \geq \hat{c}^2,\ \hat{c}^4 \geq \hat{c}^2,\ \hat{c}^1 \geq \hat{c}^4\end{array}$$

where consumption equivalents $\hat{c}_i$ are defined by

$$\begin{aligned} u(\hat{c}^1) &= u(c^1)+v(1) \iff \hat{c}^1 = u^{-1}\left(u(c^1)+v(1)\right) \\ u(\hat{c}^2) &= u(c^2) \iff \hat{c}^2 = c^2 \\ U(\hat{c}^4) &= U(c^4) \iff \hat{c}^4 = c^4 \end{aligned}$$

At the optimum, the egalitarian constraints are binding so that

$$\hat{c}^1 = \hat{c}^2 \implies c^2 > c^1 \text{ and } \hat{c}^2 = \hat{c}^4 \implies c^2 = c^4.$$

Under R2, and once type 3 has disappeared, the worst-off individuals are type-2 individuals (see Section 3), so that the problem of the planner is:

$$\begin{array}{rl}\max\limits_{c^1,c^2,c^4} & \tilde{c}^2 \\ & \text{s.t. } (\pi x+\varepsilon(1-x))c_1+(x(1-\pi)-\varepsilon(1-x))c_2+(1-x)c_4+(\pi x+\varepsilon(1-x))g \\ = & (\pi x+\varepsilon(1-x))w(1-q)+(x(1-\pi)+(1-\varepsilon)(1-x))w \\ & \text{s.t. } \tilde{c}^1 \geq \tilde{c}^2,\ \tilde{c}^4 \geq \tilde{c}^2,\ \tilde{c}^1 \geq \tilde{c}^4\end{array}$$

where equivalent consumptions $\tilde{c}^i$ are given by:

$$\begin{aligned} u\left(\tilde{c}^1\right)+v(1) &= u(c^1)+v(1) \iff \tilde{c}^1 = c^1 \\ u\left(\tilde{c}^2\right)+v(1) &= u(c^2) \iff \tilde{c}^2 = u^{-1}\left[u(c^2)-v(1)\right] \\ U(\tilde{c}^4)-V(1) &= U(c^4) \iff \tilde{c}^4 = U^{-1}\left[U(c^4)+V(1)\right] \end{aligned}$$

At the optimum, the egalitarian constraints are binding so that

$$\tilde{c}^1 = \tilde{c}^2 \implies c^2 > c^1 \text{ and } \tilde{c}^4 = \tilde{c}^1 \implies c^1 > c^4.$$

Under R3, and once type 3 has disappeared, the worst-off are type-2 individuals (see Section 3), so that the social planner's problem is:

$$\begin{array}{rl}\max\limits_{c^1,c^2,c^4} & \bar{c}^2 \\ & \text{s.t. } (\pi x+\varepsilon(1-x))c_1+(x(1-\pi)-\varepsilon(1-x))c_2+(1-x)c_4+(\pi x+\varepsilon(1-x))g \\ = & (\pi x+\varepsilon(1-x))w(1-q)+(x(1-\pi)+(1-\varepsilon)(1-x))w \\ & \text{s.t. } \bar{c}^1 \geq \bar{c}^2,\ \bar{c}^4 \geq \bar{c}^2,\ \bar{c}^1 \geq \bar{c}^4\end{array}$$

where equivalent consumptions $\bar{c}^i$ are given by:

$$\begin{aligned} u\left(\bar{c}^1\right)+v(1) &= u(c^1)+v(1) \iff \bar{c}^1=c^1 \\ u\left(\bar{c}^2\right)+v(1) &= u(c^2) \iff \bar{c}^2=u^{-1}\left[u(c^2)-v(1)\right] \\ U(\bar{c}^4) &= U(c^4) \iff \bar{c}^4=c^4 \end{aligned}$$

The egalitarian constraints are binding so that

$$\bar{c}^1=\bar{c}^2 \implies c^2>c^1 \text{ and } \bar{c}^1=\bar{c}^4 \implies c^1=c^4.$$

Under R4, and once type 3 has disappeared, the worst-off are type-2 (see Section 3), so that the problem of the planner is:

$$\begin{aligned} \max_{c^1,c^2,c^4} \quad & \breve{c}^2 \\ & \text{s.t. } (\pi x+\varepsilon(1-x))c_1+(x(1-\pi)-\varepsilon(1-x))c_2+(1-x)c_4+(\pi x+\varepsilon(1-x))g \\ = \quad & (\pi x+\varepsilon(1-x))w(1-q)+(x(1-\pi)+(1-\varepsilon)(1-x))w \\ & \text{s.t. } \breve{c}^2\geq\breve{c}^1,\ \breve{c}^4\geq\breve{c}^2,\ \breve{c}^4\geq\breve{c}^1 \end{aligned}$$

where equivalent consumptions $\breve{c}^i$ are given by:

$$\begin{aligned} u(\breve{c}^1) &= u(c^1)+v(1) \iff \breve{c}^1=u^{-1}\left(u(c^1)+v(1)\right) \\ u(\breve{c}^2) &= u(c^2) \iff \breve{c}^2=c^2 \\ U(\breve{c}^4)-V(1) &= U(c^4) \iff \breve{c}^4=U^{-1}\left[U(c^4)+V(1)\right] \end{aligned}$$

Again, when the egalitarian constraints are binding, we have

$$\breve{c}^1=\breve{c}^2 \implies c^2>c^1 \text{ and } \breve{c}^2=\breve{c}^4 \implies c^2>c^4.$$

Excess supply of children In that case, the economy is, after reallocation of children, composed of voluntary parents in proportion x, involuntary parents in proportion $(1-x)\varepsilon-(1-\pi)x$ and childfree individuals $(1-x)(1-\varepsilon)+(1-\pi)x$. The resource constraint becomes:

$$\begin{aligned} & xc_1+(\varepsilon(1-x)-(1-\pi)x)c_3+((1-x)(1-\varepsilon)+(1-\pi)x)c_4+(\pi x+\varepsilon(1-x))g \\ = \quad & (\pi x+\varepsilon(1-x))w(1-q)+((1-\varepsilon)(1-x)+(1-\pi)x)w \end{aligned}$$

The problem of the social planner consists in maximizing the consumption equivalent of the worst-off individual, taking into account that there remain only 3 types of individuals and that the worst-off individual depends on the reference fertility level. Using the findings of Section 3 and applying the same reasoning as under excess demand, one can easily show that

- Under R1, the worst-off individuals is a type-3. The optimal allocation is therefore $c^3>c^4>c^1$ so that $\hat{c}^1=\hat{c}^3=\hat{c}^4$.

- Under R2, the worst-off individuals are a type-3 or type-1 individual (since there is no type-2 individual anymore). The optimal allocation is therefore $c^1 = c^3 > c^4$ so as to obtain $\tilde{c}^1 = \tilde{c}^3 = \tilde{c}^4$.
- Under R3, the worst-off individual is a type-3 individual (since there is no type-2 individual anymore). The optimal allocation is therefore $c^3 > c^1 = c^4$ so as to obtain $\bar{c}^1 = \bar{c}^3 = \bar{c}^4$.
- Under R4, the worst-off individual is a type-3 individual. The optimal allocation should therefore be $c^1, c^4 < c^3$ so as to obtain $\breve{c}^1 = \breve{c}^3 = \breve{c}^4$.

Equal supply and demand for children The economy is, after reallocation of children, composed of voluntary parents in proportion x and of childfree individuals $(1-x)$. The resource constraint becomes:

$$xc_1 + (1-x)c_4 + xg = xw(1-q) + (1-x)w$$

Noticing that this case is a special the excess demand case where type-2 have disappeared or of the excess-supply casde where type-3 individuals have disappeared, we obtain the following socially optimal allocation. Under R1, the optimal allocation is $c^4 > c^1$. Under R2, the optimal allocation is therefore $c^1 > c^4$. Under R3, the optimal allocation is therefore $c^1 = c^4$. Under R4, the optimal allocation is $c^1 \gtreqless c^4$.

9.2 Proof of Proposition 3

In order to decentralize the first best optimum, let us consider 4 lump-sum transfers b_i given to our four types $i = \{1, 2, 3, 4\}$ of individuals. b_1 and b_3 are a child allowance to voluntary and involuntary parents respectively, while b_2 and b_4 are a transfer to involuntary and voluntary childless families.

Let us first consider the case of excess demand. After the adoption agency has reallocated children, there remains only three types of individuals: voluntary parents who receive b_1, involuntary childless with b_2 and voluntary childless with b_4. These transfers satisfy the following budget constraint.

$$b_1(x\pi + \varepsilon(1-x)) + (x(1-\pi) - \varepsilon(1-x))b_2 + (1-x)b_4 = 0. \tag{11}$$

Under R1, the decentralization of the optimum requires that the ranking of consumptions is such that $c^{4,D} = c^{2,D} > c^{1,D}$ where D stands for Decentralization, so as to reproduce the first-best optimum, $\hat{c}^1 = \hat{c}^2 = \hat{c}^4$. The lump sum transfers, b_i should satisfy:

$$\begin{aligned} u(w(1-q) - g + b_1) + v(1) &= u(w + b_2) && (12) \\ w + b_2 &= w + b_4 && (13) \end{aligned}$$

The first equation ensures that $c^{1,D} = w(1-q) - g + b_1$ and $c^{2,D} = w + b_2$ are set such that $\hat{c}^1 = \hat{c}^2$. The second equality ensures that $\hat{c}^2 = \hat{c}^4$. Obviously equation

(13) yields that $b_2 = b_4 = b$. Using equation (12) together with Assumption 1, we obtain that $b_1 < b$. For the budget constraint (11) to be satisfied, only one solution is possible: $b_1 < 0 < b$.

Under R2, the ranking of consumptions at the decentralized optimum should be that $c^{2,D} > c^{1,D} > c^{4,D}$ so as to ensure the first-best optimum is attained and $\tilde{c}^1 = \tilde{c}^2 = \tilde{c}^4$. Transfers should satisfy:

$$\begin{aligned} u(w(1-q) - g + b_1) + v(1) &= u(w + b_2) \qquad (14) \\ w(1-q) - g + b_1 &= U^{-1}[U(w + b_4) + V(1)] \qquad (15) \end{aligned}$$

so that there is full equalization of the consumption equivalents. Eq. (14) yields that $b_1 < b_2$ and eq. (15) that $b_4 < b_1$. Together with the budget constraint, two sets of solutions $b_4 < 0 < b_1 < b_2$ and $b_4 < b_1 < 0 < b_2$ are possible, depending on $u(.), U(.), v(.), V(.)$ and on parameter values.

Under R3, the ranking of consumptions should be that $c^{1,D} = c^{4,D} < c^{2,D}$ so as to ensure the first-best optimum is attained and $\bar{c}^1 = \bar{c}^2 = \bar{c}^4$. Therefore, transfers b_1, b_2 and b_4 satisfy:

$$\begin{aligned} w(1-q) - g + b_1 &= w + b_4 \implies b_1 > b_4 \\ u(w(1-q) - g + b_1) + v(1) &= u(w + b_2) \implies b_1 < b_2 \end{aligned}$$

Together with the budget constraint, only two solutions are possible: $b_4 < 0 < b_1 < b_2$ or $b_4 < b_1 < 0 < b_2$, depending on the specific forms of $u(.), U(.), v(.), V(.)$ and on parameter values.

Finally, under R4, transfers are set such as to ensure that $c^{1,D}, c^{4,D} < c^{2,D}$ and that the first-best optimum, $\breve{c}^1 = \breve{c}^2 = \breve{c}^4$, is attained. This requires:

$$\begin{aligned} u(w(1-q) - g + b_1) + v(1) &= u(w + b_2) \\ w + b_2 &= U^{-1}[U(w + b_4) + V(1)] \end{aligned}$$

so that $b_2 > b_4$ and $b_1 < b_2$ under Assumption 1. However b_1 and b_4 are not directly comparable and it depends on $u(.), U(.), v(.), V(.)$ and on parameter values. Together with the budget constraint, it leaves us with 3 possible solutions:

$$b_4, b_1 < 0 < b_2 \text{ or } b_1 < 0 < b_4 < b_2 \text{ or } b_4 < 0 < b_1 < b_2.$$

Let us now derive the ranking of the monetary transfers when there is excess supply. In that situation, group-2 individuals all become voluntary parents (type-1 individuals) while some type-3 individuals remain involuntary parents. After that reallocation of children, only 3 types of transfers are thus needed, b_1, b_3, b_4, which satisfy the following budget constraint:

$$xb_1 + (\varepsilon(1-x) - (1-\pi)x)b_3 + ((1-x)(1-\varepsilon) + (1-\pi)x)b_4 = 0$$

Following the same reasoning as in the excess demand case, we obtain the following results.

Under R1, lump-sum transfers, b_i should then satisfy the following equalities:

$$\begin{aligned} U(w(1-q)-g+b_3)-V(1) &= U(w+b_4) \\ w+b_4 &= u^{-1}[u(w(1-q)-g+b_1)+v(1)] \end{aligned}$$

This leads to $b_4 < b_3$, and $b_4 > b_1$ under Assumption 1. Together with the budget constraint of the government, we have $b_1 < 0 < b_4 < b_3$ or $b_1 < b_4 < 0 < b_3$.

Under R2, transfers should satisfy the following equalities:

$$\begin{aligned} w(1-q)-g+b_1 &= U^{-1}[U(w+b_4)+V(1)] \\ w(1-q)-g+b_1 &= w(1-q)-g+b_3 \end{aligned}$$

Together with the budget constraint, $b_4 < 0 < b_3 = b_1$ is the unique solution.

Under R3, lump-sum transfers should be such that

$$\begin{aligned} w(1-q)-g+b_1 &= w+b_4 \\ w(1-q)-g+b_1 &= U^{-1}(U(w(1-q)-g+b_3)-V(1)) \end{aligned}$$

which unambiguously yields that $b_3 > b_1 > b_4$. Together with the budget constraint, only two solutions are possible: either $b_4 < b_1 < 0 < b_3$ or $b_4 < 0 < b_1 < b_3$. The sign of b_1 depends on $u(.), U(.), v(.), V(.)$ and on parameter values.

Under R4, lump-sum transfers satisfy

$$\begin{aligned} U(w(1-q)-g+b_3)-V(1) &= U(w+b_4) \\ u^{-1}(u(w(1-q)-g+b_1)+v(1)) &= w(1-q)-g+b_3 \end{aligned}$$

which imply that $b_3 > b_4$ and $b_1 < b_3$ but b_3 and b_4 are not further comparable. Together with the budget constraint, it leaves us 3 possible solutions

$$b_4, b_1 < 0 < b_3 \text{ or } b_1 < 0 < b_4 < b_3 \text{ or } b_4 < 0 < b_1 < b_3$$

Depending on the specific forms of $u(.), U(.), v(.), V(.)$ and on parameter values, b_1 and b_4 can be positive or negative.

When there is perfect equality between supply and demand of children, there remains only two categories of individuals, voluntary parents who receive a lump sum transfer b_1 and voluntary childfree individuals who receive b_4. These lump sum transfers satisfy the following budget constraint:

$$xb_1 + (1-x)b_4 = 0$$

For each reference fertility level, the ranking of the b_is correspond to a simplification of the excess demand case where $b_2 = 0$ or of the excess supply case where $b_3 = 0$. The solutions are the following. Under R1, $b_1 < 0 < b_4$. Under R2, $b_4 < 0 < b_1$. Under R3, $b_4 < 0 < b_1$. Under R4, $b_1 < 0 < b_4$ or $b_4 < 0 < b_1$.

9.3 Proof of Proposition 4

Assume quasi-linear preferences:

$$u(x) = x \text{ and } U(x) = x - \alpha$$

Take Proof 9.2 of Proposition 3. We only consider the case of excess demand and reference fertility levels R1 and R2.

Let us first consider case R1. Using the budget constraint and the equality in transfers $b_2 = b_4 = b$, we define b as

$$b = \frac{-\left[x\pi + \varepsilon(1-x)\right] b_1}{\left[x(1-\pi) + (1-x)(1-\varepsilon)\right]}.$$

This defines locus A in the (b_1, b) space. This is a decreasing line equal to 0 at $b_1 = 0$.

Using condition (12) , we obtain locus B, defined by

$$b \quad = \quad -qw - g + b_1 + v(1)$$

This is a 45° line with strictly positive value at $b_1 = 0$ since under Assumption 1, $-qw - g + v(1) > 0$. We also have $b = 0$ at $b_1 = qw + g - v(1) < 0$.

The decentralization by a mixed adoption-transfer scheme, is possible only the two loci A and B intersect. On the contrary, the decentralization does not exist when the two loci do not intersect.

It is straightforward to see that, when an intersection takes place between locus A and B, it must be for $b_1 < 0, b > 0$. In addition, consumptions cannot be negative, so that $-\left[w(1-q) - g\right] < b_1$ and $b > -w$.

If locus B is above locus A at $b_1 = -\left[w(1-q) - g\right]$, then the decentralization of the optimum through the mixed system does not hold. That case arises when:

$$\begin{aligned} -qw - g - \left[w(1-q) - g\right] + v(1) &> \frac{-\left[x\pi + \varepsilon(1-x)\right]\left[-\left[w(1-q) - g\right]\right]}{\left[x(1-\pi) + (1-x)(1-\varepsilon)\right]} \\ \iff w &< \frac{g\left[x\pi + \varepsilon(1-x)\right] + v(1)\left[x(1-\pi) + (1-x)(1-\varepsilon)\right]}{1 - q\varepsilon(1-x) - qx\pi} \equiv \tilde{w}_1 \quad (16) \end{aligned}$$

Thus only when $w \geq \tilde{w}_1$, the mixed system can decentralize the social optimum.

Consider now case R2. The transfers must now take the following form. Using eq. (11), we obtain

$$b_4 = -b_1 \frac{\left[x\pi + (1-x)\varepsilon\right]}{(1-x)} - \frac{\left[x(1-\pi) - (1-x)\varepsilon\right]}{(1-x)} b_2 \tag{17}$$

Also, under the assumption of quasi-linearity, eq. (14) can be rewritten as

$$b_2 = -wq - g + b_1 + v(1)$$

which defines the Locus I in the (b_1, b_2) space. It is a 45 degree line which crosses the x axis at $b_1 = wq + g - v(1) < 0$.

Also replacing eq. (17) into eq. (15), we obtain after some rearrangements that

$$\begin{aligned} w(1-q)-g+b_1 &= w-b_1\frac{[x\pi+(1-x)\varepsilon]}{(1-x)}-\frac{[x(1-\pi)-(1-x)\varepsilon]}{(1-x)}b_2+V(1) \\ \rightarrow b_2 &= -b_1\frac{1-x(1-\pi)+(1-x)\varepsilon}{[x(1-\pi)-(1-x)\varepsilon]}+\frac{(1-x)\left(V(1)+wq+g\right)}{[x(1-\pi)-(1-x)\varepsilon]} \end{aligned}$$

That equation defines Locus II. It is a decreasing line that crosses the x axis at a positive level of b_1.

Note that consumption cannot be negative, so that $-(w(1-q)-g)<b_1$. Moreover $b_2>-w$. Hence the transfer system does not decentralize the optimum when the locus I remains above the locus II at $b_1=-(w(1-q)-g)$.That condition is satisfied when

$$w<\frac{g\left(x\pi+(1-x)\varepsilon\right)-(1-x)V(1)+v(1)\left[x(1-\pi)-(1-x)\varepsilon\right]}{1-(1-x)\varepsilon q-qx\pi}\equiv\tilde{w}_2 \quad (18)$$

We thus have that the mixed system decentralizes the social optimum when $w>\tilde{w}_2$ and not otherwise.

Let us finally show that $\tilde{w}_1>\tilde{w}_2$. We do so by comparing the RHS of (16) and (18) and acknowledging that $v(1)>0>-V(1)$.

9.4 Proof of Proposition 6

The constrained optimum Under R1, and using results of Section 3, the worst-off is type 3, so that the problem is:

$$\begin{aligned} \max_{c^1,c^2,c^3,c^4} \quad & \hat{c}^3 \\ & \text{s.t. } x\pi c_1+x(1-\pi)c_2+(1-x)\varepsilon c_3+(1-x)(1-\varepsilon)c_4+\pi xg+\varepsilon(1-x)g \\ = \quad & \pi xw(1-q)+(1-\pi)xw+\varepsilon(1-x)w(1-q)+(1-\varepsilon)(1-x)w \\ & \text{s.t. } \hat{c}^1\geq\hat{c}^3,\ \hat{c}^2\geq\hat{c}^3,\ \hat{c}^4\geq\hat{c}^3,\ \hat{c}^2\geq\hat{c}^1 \end{aligned}$$

This problem can be solved by first assuming that the egalitarian constraints are binding. We then have

$$\hat{c}^1=\hat{c}^2 \implies c^2>c^1 \text{ and } \hat{c}^4=\hat{c}^3 \implies c^3>c^4 \text{ and } \hat{c}^2=\hat{c}^4 \implies c^2=c^4$$

At the egalitarian optimum, if the reference fertility is zero, one should therefore implement:

$$c^3>c^4=c^2>c^1,$$

so that $\hat{c}^1=\hat{c}^2=\hat{c}^3=\hat{c}^4$.

A similar type of proof by be carried out for cases R2, R3 and R4. The following Lemma summarizes our results.

Lemma 1 *At the ex post egalitarian optimum, equivalent consumption levels are equalized for the four types, and we have:*

reference fertility	*consumption ranking*
R1 ($\bar{n}=0$)	$c^1 < c^2 = c^4 < c^3$
R2 ($\bar{n}=1$)	$c^4 < c^1 = c^3 < c^2$
R3 ($\bar{n}_{1,2}=1$, $\bar{n}_{3,4}=0$)	$c^1 = c^4 < c^2, c^3$
R4 ($\bar{n}_{1,2}=0$, $\bar{n}_{3,4}=1$)	$c^1, c^4 < c^2 = c^3$

Proof. See above. ■

Decentralization Consider now the decentralization of that constrained optimum. In order to decentralize the constrained optimum, we consider 4 monetary transfers b_i given to our four types $i = \{1, 2, 3, 4\}$ of individuals. These transfers have to satisfy the following budget constraint:

$$b_1 x\pi + x(1-\pi)b_2 + b_3(1-x)\varepsilon + (1-x)(1-\varepsilon)b_4 = 0 \tag{19}$$

Under R1, the decentralization of the optimum requires that consumptions are such that $c^{3,D} > c^{4,D} = c^{2,D} > c^{1,D}$ where D stands for Decentralization, so as to reproduce the first-best optimum, $\hat{c}^1 = \hat{c}^2 = \hat{c}^3 = \hat{c}^4$. Lump sum transfers, b_i should then satisfy:

$$\begin{aligned} u(w(1-q) - g + b_1) + v(1) &= u(w + b_2) && (20) \\ U(w(1-q) - g + b_3) - V(1) &= U(w + b_4) && (21) \\ w + b_2 &= w + b_4 && (22) \end{aligned}$$

The first equation ensures that $c^{1,D} = w(1-q) - g + b_1$ and $c^{2,D} = w + b_2$ are set such that $\hat{c}^1 = \hat{c}^2$. The second equality ensures that $c^{3,D} = w(1-q) - g + b_3$ and $c^{4,D} = w + b_4$ are set such that $\hat{c}^3 = \hat{c}^4$, while the third equality ensures that $\hat{c}^2 = \hat{c}^4$. Together with (20) and (21), this implies that $\hat{c}^1 = \hat{c}^3$ so that we have full equalization of the consumption equivalents. The last equation ensures that the government resource constraint is satisfied. Obviously equation (22) yields that $b_2 = b_4 = b$. Using equation (20) together with Assumption 1, we obtain that $b_1 < b$ and using equation (21), we have that $b < b_3$.
Let us now find the signs of $\{b, b_1, b_3\}$. Situations where $0 < b_i \forall i$ or $b_i < 0 \forall i$ would not be possible as they do not satisfy (19). Yet, using eq. (20), (21) and (22), both $b > 0$ or $b < 0$ are possible solutions which depend on the specific forms of $u(.), U(.), v(.), V(.)$ and on parameter values.

When the reference fertility level is one (R2), the ranking of consumptions at the decentralized optimum should be that $c^{2,D} > c^{1,D} = c^{3,D} > c^{4,D}$ so as to ensure the first-best optimum is attained and $\tilde{c}^1 = \tilde{c}^2 = \tilde{c}^3 = \tilde{c}^4$. The transfers should satisfy the following equalities:

$$u(w(1-q) - g + b_1) + v(1) = u(w + b_2) \tag{23}$$
$$U(w(1-q) - g + b_3) - V(1) = U(w + b_4) \tag{24}$$
$$w(1-q) - g + b_1 = w(1-q) - g + b_3 \tag{25}$$

The first constraint ensures that $\tilde{c}^1 = \tilde{c}^2$, while the second constraint ensures that $\tilde{c}^3 = \tilde{c}^4$. Eq. (25) ensures that $\tilde{c}^1 = \tilde{c}^3$, which together with (23) and (24)

ensures that $\tilde{c}^2 = \tilde{c}^4$ and so, that there is full equalization of the consumption equivalents. Eq. (25) trivially yields that $b_1 = b_3 = b$. Using (23) and (24), we obtain $b_4 < b < b_2$.
Let us now find the signs of $\{b, b_2, b_4\}$. As in the previous case, one can show that both solutions $b_4 < 0 < b < b_2$ and $b_4 < b < 0 < b_2$ are possible, depending on the specific forms of $u(.), U(.), v(.), V(.)$ and on parameter values.

A similar type of proof can be used for cases R3 and R4. The following lemma 2 summarizes our results.[44]

Lemma 2 *Assume that the economy is sufficiently productive (i.e. w is sufficiently large). The decentralization of the ex post egalitarian optimum can be achieved by means of the following instruments:*

Reference fertility	*Monetary transfers*
R1 ($\bar{n} = 0$)	$b_1 < 0 < b_2 = b_4 < b_3$ *or* $b_1 < b_2 = b_4 < 0 < b_3$
R2 ($\bar{n} = 1$)	$b_4 < 0 < b_3 = b_1 < b_2$ *or* $b_4 < b_3 = b_1 < 0 < b_2$
R3 ($\bar{n}_{1,2} = 1$, $\bar{n}_{3,4} = 0$)	$b_4 < b_1 < 0 < b_2, b_3$ *or* $b_4 < 0 < b_1 < b_2, b_3$
R4 ($\bar{n}_{1,2} = 0$, $\bar{n}_{3,4} = 1$)	$b_4, b_1 < 0 < b_2 < b_3$ *or* $b_1 < 0 < b_4 < b_2 < b_3$ *or* $b_4 < 0 < b_1 < b_2 < b_3$

Proof. See above. ■

The quasi-linear case Let us first consider the decentralization in case R1. Transfers b_1, b_2, b_3, b_4 should satisfy equations (19)-(22), where we have replaced for the quasi-linear utilities.

Obviously, eq. (22) leads to $b_2 = b_4 = b$ so that the budget constraint leads to:

$$b_3 = \frac{-x\pi b_1 - [(1-x)(1-\varepsilon) + x(1-\pi)]\, b}{(1-x)\varepsilon}$$

Eq. (20) leads to

$$b = -qw - g + b_1 + v(1)$$

This defines the locus I, i.e. the set of pairs (b_1, b). This is given by an increasing line, with slope 1 and with a positive intercept (when $b_1 = 0$) at $v(1) - qw - g > 0$ (assumption A1).

Eq. (21) together with the budget constraint

$$b = -\frac{x\pi}{1-\pi x} b_1 - \frac{V(1) + qw + g}{\frac{1-\pi x}{\varepsilon(1-x)}}$$

It defines the locus II, i.e. the set of pairs (b_1, b) such that $\hat{c}_3 = \hat{c}_4$ and the budget constraint is satisfied. Since $\pi x < 1$, it has a negative slope, less than the 45°line. When $b_1 = 0$, we have $b = -\frac{V(1)+wq+g}{\frac{1-\pi x}{\varepsilon(1-x)}} < 0$.

[44] When the sign of the transfer is ambiguous, its sign depends on the specific forms of $u(.), U(.), v(.), V(.)$ and on parameter values.

In addition, consumptions cannot be negative, so that $-[w(1-q)-g] < b_1$ and $b > -w$.

Non-existence arises if and only if locus I is above locus II at $b_1 = -[w(1-q)-g]$, that is, when:

$$w < \frac{g\left(x\pi + \varepsilon(1-x)\right) + v(1)\left(1-\pi x\right) + V(1)\varepsilon(1-x)}{1 - x\pi q - \varepsilon q(1-x)} \equiv \bar{w}_1 \tag{26}$$

Thus the decentralization through a pure transfer system exists when $w \geq \bar{w}_1$, and does not exist when $w < \bar{w}_1$.

Consider now case R2. The transfers should satisfy the budget constraint (19) as well as equations (23)-(25) where we assume quasi-linear utilities . Eq. (25) obviously leads to $b_1 = b_3 = b$ and eq. (23) to

$$b_2 = b - wq - g + v(1).$$

This defines locus I in the (b, b_2) space. Locus I is a 45 degree line crossing the x axis for a negative value of $b = wq + g - v(1)$.

Using the budget constraint, on can rewrite b_4 as

$$b_4 = \frac{-b\left(x\pi + (1-x)\varepsilon\right) - x(1-\pi)b_2}{(1-x)(1-\varepsilon)}$$

and replacing it in (24), we obtain

$$b_2 = -\frac{1-x+x\pi}{x(1-\pi)}b + \frac{(1-x)(1-\varepsilon)}{x(1-\pi)}\left(V(1) + wq + g\right)$$

This defines locus II in the (b, b_2) space. Locus II is a decreasing line crossing the x axis at a positive value for b.

Note that consumptions cannot be negative, so that $c_1 > 0$ implies $b \geq -w(1-q) + g$. Moreover, $c_2 > 0$ implies $b_2 \geq -w$.

Decentralization cannot take place when the locus I is above the locus II at $b = (-w(1-q) + g)$. That condition can be written as:

$$w < \frac{g\left(x\pi + \varepsilon(1-x)\right) + v(1)x(1-\pi) - (1-x)(1-\varepsilon)V(1)}{1 - qx\pi - q\varepsilon(1-x)} \equiv \bar{w}_2 \tag{27}$$

Thus the decentralization of the constrained optimum through a transfer system is possible only if $w \geq \bar{w}_2$.

Comparing the RHS of (26) with (27), it straightforward to show that $\bar{w}_1 > \bar{w}_2$, since $v(1) > 0 > -V(1)$.

In the same way, it is straightforward to show that $\bar{w}_1 > \tilde{w}_1$ by comparing the RHS of (16) and (26), and that $\bar{w}_2 > \tilde{w}_2$ by comparing the RHS of (18) and (27) and recognizing that $v(1) > 0 > -V(1)$.

Finally comparing the RHS of (16) and (27), we obtain that $\tilde{w}_1 > \bar{w}_2$ so that we obtain the following complete ranking:

$$\bar{w}_1 > \tilde{w}_1 > \bar{w}_2 > \tilde{w}_2.$$

From these inequalities, we obtain the results of Proposition (3).

Our results are summarized in the following Lemma 3.

Lemma 3 *Assume quasi-linear preferences. Assume that there is excess demand for children, i.e.* $\varepsilon < \frac{x(1-\pi)}{(1-x)}$. *Define*

$$\begin{aligned}\bar{w}_1 &\equiv \frac{g\left(x\pi + \varepsilon(1-x)\right) + v(1)\left(1-\pi x\right) + V(1)\varepsilon(1-x)}{1-\varepsilon q(1-x) - qx\pi} \\ \bar{w}_2 &\equiv \frac{g\left(x\pi + \varepsilon(1-x)\right) + v(1)x(1-\pi) - (1-x)(1-\varepsilon)V(1)}{1-\varepsilon q(1-x) - qx\pi}\end{aligned}$$

Threshold hourly wage levels are ranked as follows:

$$\tilde{w}_2 < \bar{w}_2 < \tilde{w}_1 < \bar{w}_1$$

where $\tilde{w}_1$ *and* $\tilde{w}_2$ *are defined in Proposition 4.*

- *Under R1* ($\bar{n} = 0$),
  - *if* $w > \bar{w}_1 > \tilde{w}_1$, *the equalization of* $\hat{c}^i$*s can be achieved by either a pure transfer system or by a mixed system.*
  - *if* $\bar{w}_1 > w > \tilde{w}_1$, *the equalization of* $\hat{c}^i$*s can only be achieved by a mixed system.*
  - *if* $\bar{w}_1 > \tilde{w}_1 > w$, *the equalization of* $\hat{c}^i$*s cannot be achieved.*
- *Under R2* ($\bar{n} = 1$),
  - *if* $w > \bar{w}_2 > \tilde{w}_2$, *the equalization of* $\tilde{c}^i$*s can be achieved by either a pure transfer system or by a mixed system.*
  - *if* $\bar{w}_2 > w > \tilde{w}_2$, *the equalization of* $\tilde{c}^i$*s can only be achieved by a mixed system.*
  - *if* $\bar{w}_2 > \tilde{w}_2 > w$, *the equalization of* $\tilde{c}^i$*s cannot be achieved.*

Proof. See above. ■

9.5 Proof of Proposition 8

Under R1 ($\bar{n} = 0$ and $\bar{\ell} = 0$ for all i), equivalent consumption levels satisfy:

$$\begin{aligned}u\left(\hat{c}^1\right) &= u\left(c^1\right) + v(1) \implies \hat{c}^1 = u^{-1}\left[u\left(c^1\right) + v(1)\right] \\ u\left(\hat{c}^2\right) &= u(c^2) + v(1) - \varphi\left(\ell\right) \implies \hat{c}^2 = u^{-1}\left[u(c^2) + v(1) - \varphi\left(\ell\right)\right] \\ u\left(\hat{c}^3\right) &= u(c^3) - \varphi\left(\ell\right) \implies \hat{c}^3 = u^{-1}\left[u(c^3) - \varphi\left(\ell\right)\right] \\ U(\hat{c}^4) &= U(c^4) \implies \hat{c}^4 = c^4\end{aligned}$$

At the egalitarian optimum, egalitarian constraints are binding. We have

$$\hat{c}^1 = \hat{c}^2 \implies c^1 < c^2 \text{ and } \hat{c}^4 = \hat{c}^3 \implies c^4 < c^3 \text{ and } \hat{c}^4 = \hat{c}^2 \implies c^2 < c^4$$

Note that $\hat{c}^4 = \hat{c}^2$ is obtained recognizing that $v(1) - \varphi\left(\ell\right) > 0$ (See Section 6.1).

Under R2 ($\bar{n} = 1$ and $\bar{\ell} = 0$ for all i), equivalent consumption levels satisfy:

$$\begin{aligned}
u\left(\tilde{c}^1\right) + v(1) &= u\left(c^1\right) + v(1) \implies \tilde{c}^1 = c^1 \\
u\left(\tilde{c}^2\right) + v(1) &= u(c^2) + v(1) - \varphi\left(\ell\right) \implies \tilde{c}^2 = u^{-1}\left[u(c^2) - \varphi\left(\ell\right)\right] \\
u\left(\tilde{c}^3\right) + v(1) &= u(c^3) - \varphi\left(\ell\right) \implies \tilde{c}^3 = u^{-1}\left[u(c^3) - \varphi\left(\ell\right) - v(1)\right] \\
U(\tilde{c}^4) - V(1) &= U(c^4) \implies \tilde{c}^4 = U^{-1}\left[U(c^4) + V(1)\right]
\end{aligned}$$

At the egalitarian optimum, if egalitarian constraints are binding, we have

$$\tilde{c}^1 = \tilde{c}^2 \implies c^1 < c^2 \text{ and } \tilde{c}^2 = \tilde{c}^3 \implies c^2 < c^3 \text{ and } \tilde{c}^1 = \tilde{c}^4 \implies c^1 > c^4$$

Under R3 ($\bar{n} = 1 \forall i$ and the reference ART effort is $\bar{e} = \ell$ for types $i = \{1, 2, 3\}$ and 0 for type 4), equivalent consumption levels satisfy:

$$\begin{aligned}
u\left(\bar{c}^1\right) + v(1) - \varphi\left(\ell\right) &= u\left(c^1\right) + v(1) \implies \bar{c}^1 = u^{-1}\left[u\left(c^1\right) + \varphi\left(\ell\right)\right] \\
u\left(\bar{c}^2\right) + v(1) - \varphi\left(\ell\right) &= u(c^2) + v(1) - \varphi\left(\ell\right) \implies \bar{c}^2 = u^{-1}\left[u\left(c^2\right)\right] = c^2 \\
u\left(\bar{c}^3\right) + v(1) - \varphi\left(\ell\right) &= u(c^3) - \varphi\left(\ell\right) \implies \bar{c}^3 = u^{-1}\left[u(c^3) - v(1)\right] \\
U(\bar{c}^4) - V(1) &= U(c^4) \implies \bar{c}^4 = U^{-1}\left[U(c^4) + V(1)\right]
\end{aligned}$$

At the egalitarian optimum, if egalitarian constraints are binding, we have

$$\begin{aligned}
\bar{c}^1 &= \bar{c}^2 \implies c^1 < c^2 \text{ and } \bar{c}^2 = \bar{c}^3 \implies c^2 < c^3 \\
\bar{c}^4 &= \bar{c}^2 \implies c^4 < c^2 \text{ and } \bar{c}^1 = \bar{c}^4 \implies c^1 \gtreqless c^4
\end{aligned}$$

9.6 Proof of Proposition 9

We define b_i as the transfers given to the i-types. These transfers must ensure that individuals who cannot have children but want one decide to invest in ART ($e = \ell$) after government's intervention, that is, the b_is must satisfy:[45]

$$p[u(w(1-q)-g-\ell+b_2)+v(1)-\varphi(\ell)]+(1-p)[u(w-\ell+b_3)-\varphi(\ell)] \geq u(w) \quad (28)$$

where the RHS is the utility the individual if he does not invest in ART and remains childless with probability 1. For each reference fertility level, we will check that this inequality is effectively satisfied.

Under R1, these transfers have to satisfy:

$$\begin{aligned}
&\hat{c}^1 = \hat{c}^2 \iff u(w(1-q) - g + b_1) + v(1) = u(w(1-q) - g - \ell + b_2) + v(1) - \varphi\left(\ell\right) \\
&\hat{c}^2 = \hat{c}^3 \iff u(w(1-q) - g - \ell + b_2) + v(1) - \varphi\left(\ell\right) = u(w - \ell + b_3) - \varphi\left(\ell\right) \\
&\hat{c}^3 = \hat{c}_4 \iff u^{-1}(u(w - \ell + b_3) - \varphi\left(\ell\right)) = U^{-1}(U(w + b_4)) \\
&\hat{c}^2 = \hat{c}^4 \iff u^{-1}(u(w(1-q) - g - \ell + b_2) + v(1) - \varphi\left(\ell\right)) = U^{-1}(U(w + b_4)) \\
&x\pi b_4 + x(1-\pi)pb_2 + x(1-p)b_3 + (1-x)b_4 = 0
\end{aligned}$$

[45] It could be the case that after the government's intervention, individuals choose not to invest anymore in ART, in which case we would be back to our original modeling with no ART. See Sections 2 to 5.

The first four lines, which ensure that $\hat{c}^1 = \hat{c}^2 = \hat{c}^3 = \hat{c}^4$, imply that $b_2 > b_1$, $b_2 < b_3$, $b_4 < b_3$, $b_2 < b_4$ and so, that $b_1 < b_2 < b_4 < b_3$. Using the budget constraint, one cannot have that $b_i > 0 \forall i$, neither that $b_i < 0 \forall i$. Under the full equalization of all consumption equivalents, condition (28) can be rewritten as $u(\hat{c}_2) = u(\hat{c}_3) \geq u(w)$ and using $u(\hat{c}_3) = u(\hat{c}_4) = u(w + b_4)$, this yields that $b_4 \geq 0$. There are thus 2 possible solutions:

$$b_1 < 0 < b_2 < b_4 < b_3 \text{ or } b_1 < b_2 < 0 \leq b_4 < b_3.$$

Under R2, transfers b_i have to satisfy:

$$\begin{aligned}
\tilde{c}_1 = \tilde{c}_2 &\iff w(1-q) - g + b_1 = u^{-1}[u(w(1-q) - g - \ell + b_2) - \varphi(\ell)] \\
\tilde{c}_2 = \tilde{c}_3 &\iff u(w(1-q) - g - \ell + b_2) - \varphi(\ell) = u(w - \ell + b_3) - \varphi(\ell) - v(1) \\
\tilde{c}_1 = \tilde{c}_4 &\iff w(1-q) - g + b_1 = U^{-1}(U(w + b_4) + V(1))
\end{aligned}$$

to ensure that the first-best optimum, $\tilde{c}^1 = \tilde{c}^2 = \tilde{c}^3 = \tilde{c}^4$ is implemented. Proceeding in the same way as in the previous case, the above equalities imply that $b_4 < b_1 < b_2 < b_3$. Using the budget constraint, one cannot have that $b_i > 0 \forall i$, neither that $b_i < 0 \forall i$. Hence only 3 solutions are possible:

$$b_4 < 0 < b_1 < b_2 < b_3 \text{ or } b_4 < b_1 < 0 < b_2 < b_3 \text{ or } b_4 < b_1 < b_2 < 0 < b_3.$$

This set of solutions is a priori compatible with condition (28) for positive investment in ART.[46]

Under R3, transfers b_i have to satisfy:

$$\begin{aligned}
\bar{c}_1 = \bar{c}_2 &\iff u^{-1}(u(w(1-q) - g + b_1) + \varphi(\ell)) = w(1-q) - g - \ell + b_2 \\
\bar{c}_2 = \bar{c}_3 &\iff u(w(1-q) - g - \ell + b_2) = u(w - \ell + b_3) - v(1) \\
\bar{c}_1 = \bar{c}_4 &\iff u^{-1}(u(w(1-q) - g + b_1) + \varphi(\ell)) = U^{-1}(U(w + b_4) + V(1)) \\
\bar{c}_2 = \bar{c}_4 &\iff w(1-q) - g - \ell + b_2 = U^{-1}(U(w + b_4) + V(1))
\end{aligned}$$

to ensure that the first-best optimum, $\bar{c}^1 = \bar{c}^2 = \bar{c}^3 = \bar{c}^4$, is implemented. The above equalities yield that $b_4, b_1 < b_2 < b_3$ and $b_4 \gtrless b_1$. Since the ranking between b_1 and b_4 is indeterminate, 6 solutions are possible:

$$\begin{aligned}
b_4 &< 0 < b_1 < b_2 < b_3;\ b_4 < b_1 < 0 < b_2 < b_3;\ b_4 < b_1 < b_2 < 0 < b_3, \\
\text{or } b_1 &< 0 < b_4 < b_2 < b_3;\ b_1 < b_4 < 0 < b_2 < b_3;\ b_1 < b_4 < b_2 < 0 < b_3,
\end{aligned}$$

This set of solutions is again compatible with the condition (28) for positive investment in ART.

[46] The exclusion of solutions would depend on the functional forms of individual preferences as well as on the value of parameters.