A Model of Secular Stagnation: Theory and Quantitative Evaluation

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Facing Demographic Change in a Challenging Economic Environment
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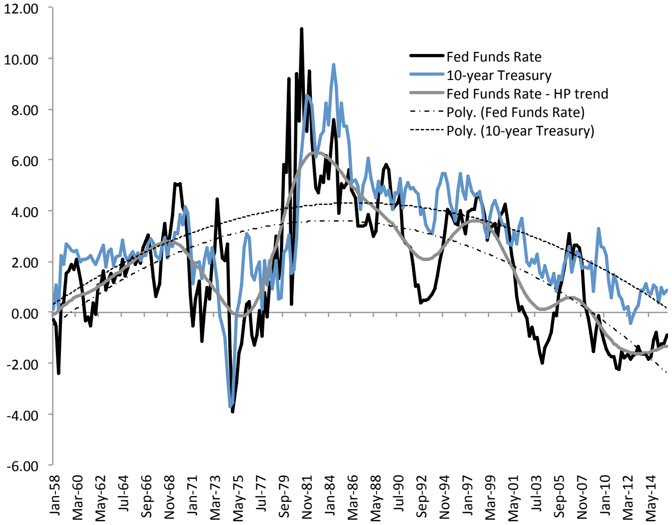
Decline in US Real Rates

US Treasuries



Research Question and Approach

Research questions:

- Can we formalize the idea of secular stagnation?
- Is the natural rate of interest negative and what accounts for the fall in the natural rate?

Approach:

- Simple three period model along the lines of Samuelson (1958)
- Quantitative lifecycle model (56 periods)

Preview of Findings

Properties of secular stagnation equilibrium:

- Slow moving factors that can push the natural rate negative
- ZLB may be binding for an arbitrarily long-period of time
- ZLB steady state is locally determinate

Quantitative findings:

- Fully account for decline in interest rates from 1970 to 2015
- Demographic factors and productivity play the largest role
- Secondary role for capital demand factors

Implications for monetary policy:

- Irrelevance of forward guidance
- Significant fall in long-run neutral interest rate to -1.5% to -2%
- Zero lower bound likely to remain binding

HOUSEHOLDS

Objective function:

$$\max_{C_{t,}^{y},C_{t+1}^{m},C_{t+2}^{o}} U = \mathbb{E}_t \left\{ \log\left(C_t^y\right) + \beta \log\left(C_{t+1}^m\right) + \beta^2 \log\left(C_{t+2}^o\right) \right\}$$

Budget constraints:

$$\begin{aligned}
C_t^y &= B_t^y \\
C_{t+1}^m &= Y_{t+1}^m - (1+r_t)B_t^y + B_{t+1}^m \\
C_{t+2}^o &= -(1+r_{t+1})B_{t+1}^m \\
(1+r_t)B_t^i &\leq D_t
\end{aligned}$$

Consumption and Saving

Credit-constrained youngest generation:

$$C_t^y = B_t^y = \frac{D_t}{1 + r_t}$$

Saving by the middle generation:

$$\frac{1}{C_t^m} = \beta \mathbb{E}_t \frac{1 + r_t}{C_{t+1}^o}$$

Spending by the old:

$$C_t^o = -(1 + r_{t-1}) B_{t-1}^m$$

Determination of the Real Interest Rate

Asset market equilibrium:

$$N_t B_t^y = -N_{t-1} B_t^m$$
$$(1 + g_t) B_t^y = -B_t^m$$

Demand and supply of loans:

$$L_t^d = \frac{1 + g_t}{1 + r_t} D_t$$
$$L_t^s = \frac{\beta}{1 + \beta} \left(Y_t^m - D_{t-1}\right)$$

Determination of the Real Interest Rate

Expression for the real interest rate:

$$1 + r_t = \frac{1+\beta}{\beta} \frac{(1+g_t)D_t}{Y_t^m - D_{t-1}}$$

Determinants of the real interest rate:

- Tighter collateral constraint reduces the real interest rate
- Lower rate of population growth reduces the real interest rate

Price Level Determination

Euler equation for nominal bonds:

$$\frac{1}{C_t^m} = \beta \mathbb{E}_t \frac{1}{C_{t+1}^o} (1 + i_t) \frac{P_t}{P_{t+1}}$$
$$i_t \geq 0$$

Bound on steady state inflation:

$$\bar{\Pi} \geq \frac{1}{1+r}$$

- If steady state real rate is negative, steady state inflation must be positive
- No equilibrium with inflation below $\bar{\Pi}$
- But what happens when prices are NOT flexible and the central bank does not tolerate inflation $\bar{\Pi}$ or higher?

Money

Endogenous Production

Production and income:

$$Y_t = L_t^{\alpha}$$

- Labor only factor of production
- Firms are perfectly competitive

Labor supply:

- Constant inelastic labor supply from households
- Assume only middle-generation household supplies labor
- Possibility of unemployment due to wage rigidity

Aggregate Supply - Full Employment

Output and labor demand:

$$Y_t = L_t^{\alpha}$$
$$\frac{W_t}{P_t} = \alpha L_t^{\alpha-1}$$

Labor supply:

- Middle-generation households supply a constant level of labor $\bar{L}$
- Implies a constant market clearing real wage $\bar{W} = \alpha \bar{L}^{\alpha-1}$
- Implies a constant full-employment level of output: $Y_{fe} = \bar{L}^{\alpha}$

Downward Nominal Wage Rigidity

Partial wage adjustment:

$$W_t = \max\left\{\tilde{W}_t, P_t \alpha \bar{L}^{\alpha-1}\right\}$$
$$\text{where } \tilde{W}_t = \gamma W_{t-1} + (1-\gamma) P_t \alpha \bar{L}^{\alpha-1}$$

Wage rigidity and unemployment:

- $\tilde{W}_t$ is a wage norm
- If real wages exceed market clearing level, employment is rationed
- Unemployment: $U_t = \bar{L} - L_t$
- Similar assumption in Kocherlakota (2013) and Schmitt-Grohe and Uribe (2013)

Steady State Aggregate Supply Relation

For positive steady state inflation:

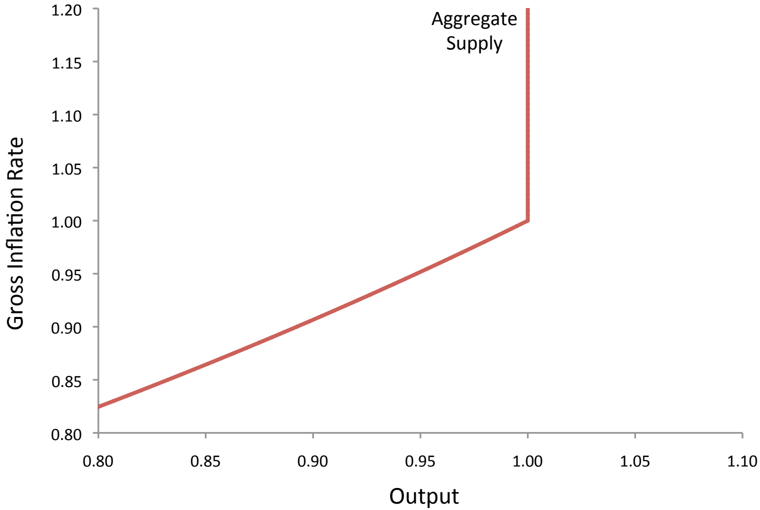
$$Y = Y_{fe} = \bar{L}^{\alpha}$$

For steady state deflation:

$$\frac{Y}{Y_{fe}} = \left(\frac{1 - \frac{\gamma}{\Pi}}{1 - \gamma} \right)^{\frac{\alpha}{1-\alpha}}$$

- Upward sloping relationship between inflation and output
- Vertical line at full-employment

Aggregate Supply Relation



Steady State Aggregate Demand Relation

Above binding ZLB:

$$\frac{1+i^*}{\Pi}\left(\frac{\Pi}{\Pi^*}\right)^{\phi_\pi} = \frac{1+\beta}{\beta}\frac{(1+g)D}{Y-D}$$

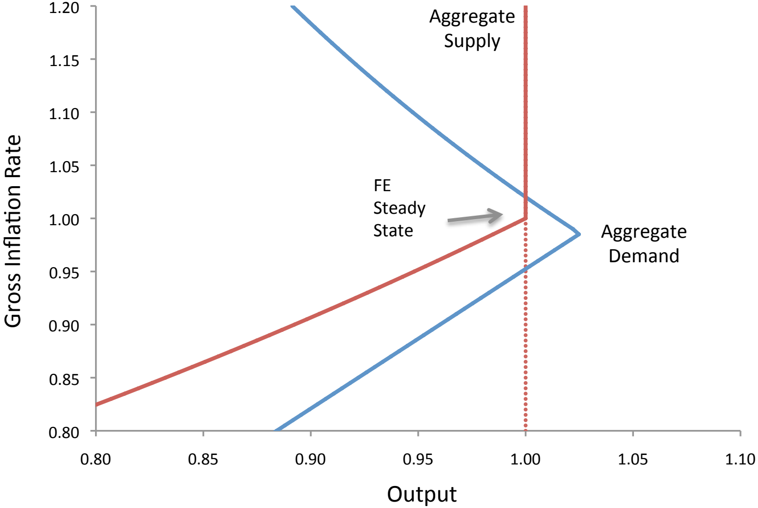
Binding ZLB:

$$\frac{1}{\Pi} = \frac{1+\beta}{\beta}\frac{(1+g)D}{Y-D}$$

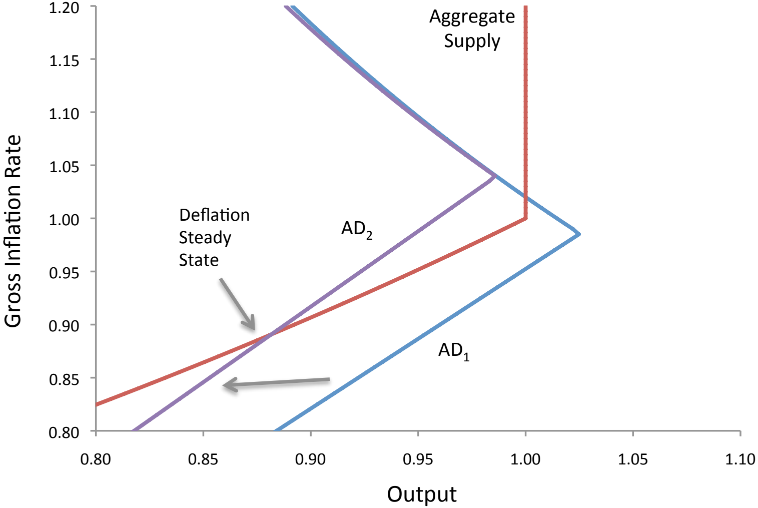
Inflation rate at which ZLB binds:

$$\Pi_{kink} = \Pi^*\left(\frac{1}{1+i^*}\right)^{\frac{1}{\phi_\pi}}$$

FULL EMPLOYMENT STEADY STATE



Effect of a Fall in the Natural Rate



Properties of the Stagnation Steady State

Long slump:

- Binding zero lower bound so long as natural rate is negative
- Deflation raises real wages above market-clearing level
- Output persistently below full-employment level

Existence and stability:

- Secular stagnation steady state exists so long as $\gamma > 0$
- If $\Pi^* = 1$, secular stagnation steady state is unique and determinate
- Contrast to deflation steady state emphasized in Benhabib, Schmitt-Grohe and Uribe (2001)

Monetary Policy Responses

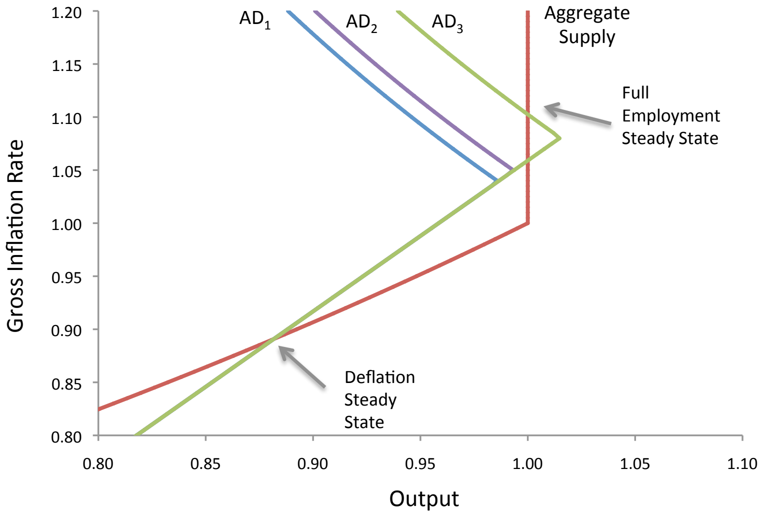
Forward guidance:

- Extended commitment to keep nominal rates low?
- Ineffective if households/firms expect rates to remain low indefinitely
- IS curve not forward-looking in the same manner as New Keynesian IS curve

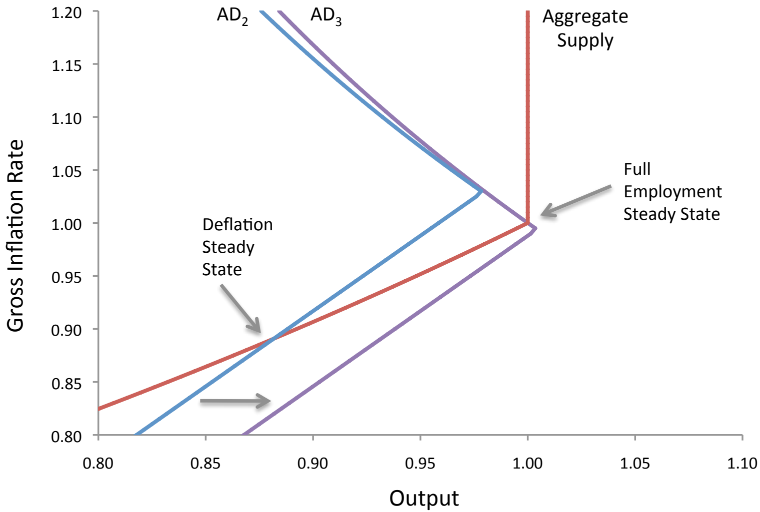
Raising the inflation target:

- For sufficiently high inflation target, full employment steady state exists.
- Timidity trap (Krugman (2014))
- Multiple determinate steady states

Raising the Inflation Target



Expansionary Fiscal Policy



Permanent Increase in Public Debt

Consider steady state following fiscal rule:

$$
\begin{aligned}
T^o &= \beta \left(1 + r\right) T^m \\
L^d &= \frac{1+g}{1+r} D + B^g \\
L^s &= \frac{\beta}{1+\beta} \left(Y^m - D\right) - \frac{1}{1+\beta} \frac{Y^o}{1+r}
\end{aligned}
$$

Implications for natural rate:

- Changes in taxation have no effects on loan supply
- Permanent rise in public debt always raises the real rate
- Public debt circumvents the tightening credit friction (Woodford (1990))

Temporary Increase in Public Debt

Under constant population and set $G_t = T_t^y = B_{t-1}^g = 0$:

$$T_t^m = -B_t^g$$
$$T_{t+1}^o = (1 + r_t)\, B_t^g$$

Implications for natural rate:

- Loan demand and loan supply effects cancel out
- Temporary increases in public debt ineffective in raising real rate
- Temporary monetary expansion equivalent to temporary expansion in public debt at the zero lower bound
- Effect of an increase in public debt depends on beliefs about future fiscal policy

Demographics

Definitions:

- $N_{j,t}$ for $j \in \{0, \ldots, J\}$ is the population at time t of a cohort of age j
- $p_{j,t+1}$ for $j \in \{0, \ldots, J\}$ is the probability of survival between t and $t+1$ for a household of age j

Population dynamics:

$$N_{j+1,t+1} = p_{j,t+1} N_{j,t}$$
$$N_{0,t+1} = n_{t+1} N_t$$
$$N_t = \sum_{j=0}^{J} N_{j,t}$$

Households

Objective function:

$$U_t = \max_{c_{j,t+j},\, a_{j+1,t+1}} \mathbb{E}_t \left(\beta^J s_{t,t+J} v\left(a_{J+1,t+J+1}\right) + \sum_{j=0}^{J} \beta^j s_{t,t+j} u\left(c_{j,t+j}\right) \right)$$

$$s_{t,t+j} = \prod_{l=0}^{j-1} p_{t+l,t+l+1}$$

Budget and debt constraints:

$$\begin{aligned} c_{j,t+j} + \zeta_{t+j} a_{j+1,t+1} &= w_{t+j} h c_j + \Pi_{j,t+j} \\ &\quad + \left(r^k_{t+j} + \zeta_{t+j}\left(1-\delta\right)\right) \left(a_{j,t+j} + q_{j,t+j} + \frac{1 - p_{j-1,t+j}}{p_{j-1,t+j}} a_{j,t+j} \right) \\ \zeta_t a_{j+1,t+1} &\geq \phi_{j,t} \\ a_{0,t} &= 0 \end{aligned}$$

Household Optimality Conditions

Household Euler equation:

$$
\begin{aligned}
\zeta_{t+j} u_c\left(c_{j,t+j}\right) &= \beta \mathbb{E}_{t+j} u_c\left(c_{j+1,t+j+1}\right)\left(r^k_{t+j+1} + \zeta_{t+j+1}\left(1-\delta\right)\right) + \mu_{j,t+j} \\
\mu_{j,t+j} &\geq 0 \\
a_{j+1,t+j+1} &\geq \phi_{j,t+j}
\end{aligned}
$$

Optimal bequest condition:

$$
\begin{aligned}
u_c\left(c_{J,t+J}\right) &= v_a\left(a_{J+1,t+J+1}\right) + \mu_{J,t+J} \\
\mu_{J,t+J} &\geq 0 \\
a_{J+1,t+J+1} &\geq 0
\end{aligned}
$$

Risk-free rate:

$$
1 + r_t = \frac{r^k_{t+1} + \left(1-\delta\right)\zeta_{t+1}}{\zeta_t}
$$

Final Goods Firms

Firms' optimization problem:

$$\max \ \frac{p_t(i)}{P_t} y_t(i) - \frac{p_t^{int}}{P_t} y_t(i)$$

$$\text{subject to } \ y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta_t}$$

Optimality condition:

$$\frac{p_t(i)}{P_t} = \frac{\theta_t}{\theta_t - 1} \frac{p_t^{int}}{P_t}$$

$$P_t = \left(\int p_t(i)^{1-\theta_t} \, di \right)^{\frac{1}{1-\theta_t}}$$

$$\Rightarrow \frac{p_t^{int}}{P_t} = \frac{\theta_t - 1}{\theta_t}$$

Intermediate Goods Firms

Firm's problem:

$$\Pi_t^{int} = \max \; \frac{p_t^{int}}{P_t} Y_t - w_t L_t - r_t^k K_t$$

$$\text{subject to } \; Y_t = \left(\alpha \left(A_{k,t} K_t\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(A_{l,t} L_t\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

Factor demands:

$$w_t = \frac{p_t^{int}}{P_t} (1-\alpha) A_{l,t}^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t}\right)^{\frac{1}{\sigma}}$$

$$r_t^k = \frac{p_t^{int}}{P_t} \alpha A_{k,t}^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}}$$

Market Clearing

Labor market clearing:

$$L_t = \sum_{j=0}^{J} N_{j,t} hc_j$$

Asset market clearing:

$$K_{t+1} = \sum_{j=0}^{J} N_{j,t} a_{j+1,t+1}$$

Distribution of bequests and profits:

$$N_{J,t-1} a_{J+1,t} = \sum_{j=0}^{J} N_{j,t} q_{j,t}$$

$$Y_t \left(1 - \frac{p_t^{int}}{P_t}\right) = \sum_{j=0}^{J} N_{j,t} \Pi_{j,t}$$

Calibration Strategy

- Set of parameters directly measured in the data (survival rates, productivity growth, etc.)
- Set of parameters taken from the literature (IES, production elasticity)
- Set of parameters chosen to match key targets
  - Interest rate in 2015
  - Match the investment to output ratio and labor share in 2015
  - Data on bequests and unsecured consumer credit
- Study both stationary equilibrium and nonlinear transition path

Calibration

Data and Related Literature

Panel A: Data	*Symbol*	*Value*	*Source*
Mortality profile	$s_{j,t}$		US mortality tables, CDC
Income profile	hc_j		Gourinchas and Parker (2002)
Total fertility rate	n	1.88	US Census Bureau
Prod. growth	g	0.65%	Fernald (2012)
Gov. spending	G	21.3%	CEA
Public debt	B_g	118%	Flow of Funds
Panel B: Related literature			
EIS	ρ	0.75	Cooley and Prescott (1996)
Production EIS	σ	0.6	Oberfield and Raval (2014)
Deprecation	δ	12%	Jorgenson (1996)

Calibration

Targeted Moments

Targets	*Model/Data*	*Source*
Natural rate of interest	-1.47%	PCE FFR
Investment to output ratio	15.9%	NIPA
Consumer debt to output ratio	6.3%	Flow of Funds
Labor share	66.0%	Elsby (2013)
Bequests to output	3.0%	Lutz (2001)
Parameters chosen to match targets	*Symbol*	*Value*
Rate of time preference	β	0.98
Borrowing limit (% of annual income)	D	23.4%
Bequests parameter	μ	21.6
Retailer elasticity of substitution	θ	4.9
Capital share parameter	α	0.24

Implications for Secular Stagnation

- If the natural rate of interest is too low relative to the inflation target, zero lower bound may be binding indefinitely

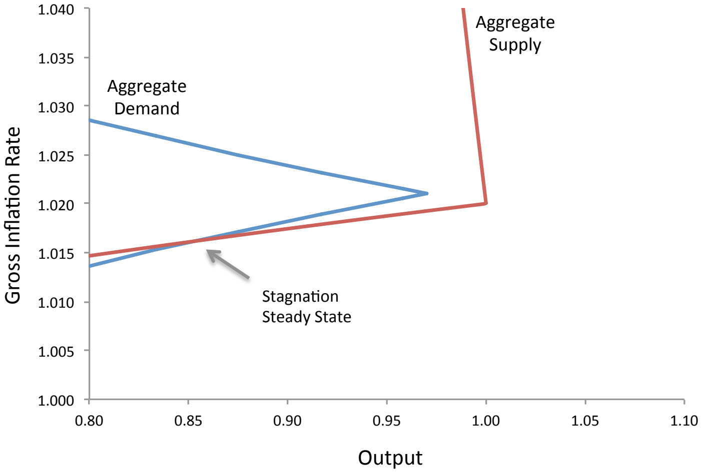
- Recalibrate model to fully explain the 15% deviation of US output from its pre-recession trend

- With inflation target $\bar{\Pi}$ is 2%, we project that neutral interest rate remains below -2% in medium term (2015-2030) implying a binding zero lower bound for foreseeable future

- In the presence of nominal wage rigidities, binding ZLB may lead to prolonged output gap

- Increased risk of zero lower bound episodes and complications for monetary policy to cushion against demand shocks

Output Gap at the ZLB



Decomposition of Fall in Interest Rates

1970 to 2015

Forcing variable	Δ in r	*% of total Δ*	*1970*	*2015*
Δ interest rate	-4.02	100%	2.55%	-1.47%
Mortality rate	-1.82	43%	70.7	78.7
Total fertility rate	-1.84	43%	2.8	1.9
Prod. growth	-1.90	44%	2.02%	0.65%
Government debt	$+2.11$	-49%	42%	118%
Labor share	-0.52	12%	72.4%	66.0%
Price of inv. goods	-0.44	10%	1.3	1.0
Change in debt limit	$+0.13$	-3%	4.21%	6.33%

Restoring a Positive Natural Rate

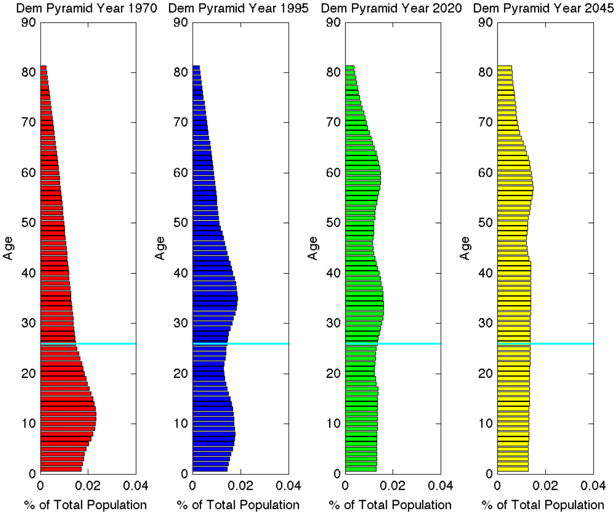
Can we raise the natural rate?

- FOMC anticipates a long-run neutral real interest rate of 1%
- What changes are needed to raise the natural rate to that level?

Forcing variable	2015	Counterfactual
Total fertility rate	1.88	3.28
Government debt (% of GDP)	118%	215%
Productivity growth	0.65%	2.43%
Relative price of investment goods	1.00	2.43

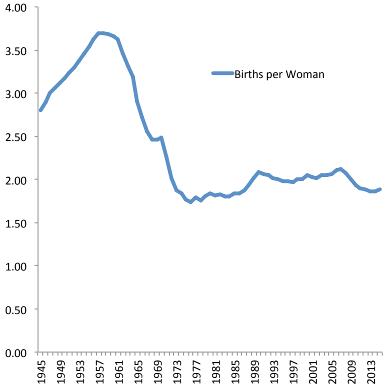
Postwar Baby Boom

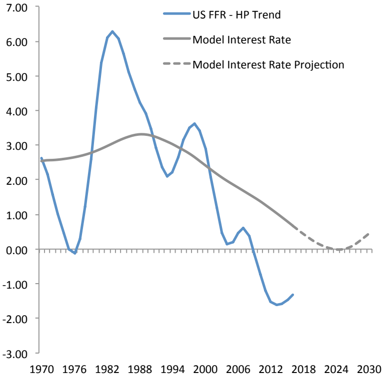
Population Distribution



Postwar Baby Boom

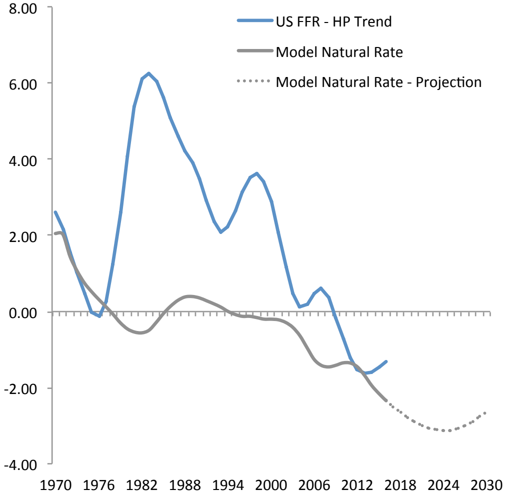
Transition Path





Combined Effect

Transition Path



I/Y and K/Y Ratio

Objections to Low Natural Rates of Interest

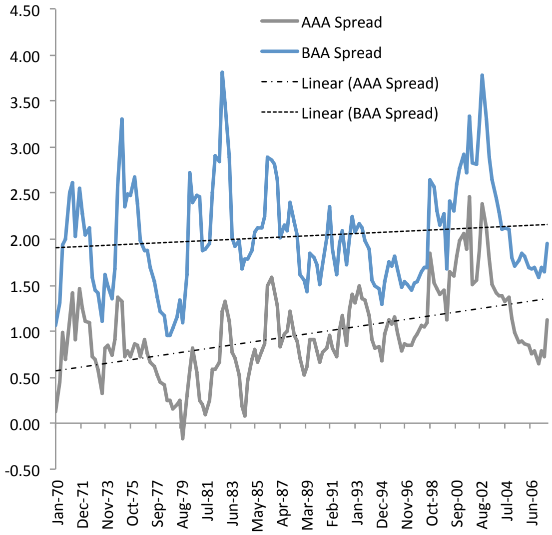
- **Objection:** Secular stagnation models may imply dynamic inefficiency
  - Typical dynamic efficiency condition is satisfied in quantitative model: $MPK - \delta > n$
  - Presence of firm monopoly powers places a wedge between MPK (1.8%) and rental rate
- **Objection:** Average product of capital fairly stable over postwar period (Gomme, Ravikumar, and Rupert (2015))
  - Average return on capital in the model falls by 60 basis points

Summary of Findings

- Locally determinate secular stagnation steady state
- Distinct policy recommendations relative to existing class of ZLB models
- Slower TFP growth and aging can explain the magnitude of the fall in real interest rates
- Can generate an estimate for the natural rate of interest that is substantially negative today and likely to remain negative in the immediate future
- Demographic factors best fit the medium-term behavior of interest rates and relative stability of I/Y

Additional Slides

AAA and BAA Spreads



Source: Federal Reserve Bank of St. Louis

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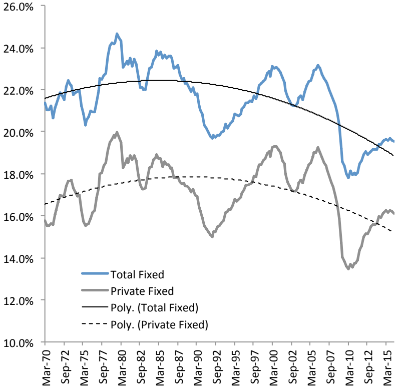
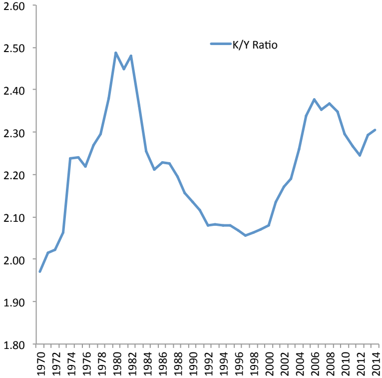
I/Y and K/Y Ratios





Source: BEA National Accounts and Fixed Asset Tables

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MONEY

Money demand condition:

$$C_t^m v'(M_t) = \frac{i_t}{1+i_t}$$

Government budget constraint:

$$B_t^g + M_t + T_t^m + \frac{1}{1+g_{t-1}} T_t^o = G_t + \frac{1}{1+g_{t-1}} \left(\frac{1+i_{t-1}}{\Pi_t} B_{t-1}^g + \frac{1}{\Pi_t} M_{t-1} \right)$$

Implications:

- Assume that money demand is satiated at the zero lower bound
- Fiscal policy keeps *real* government liabilities constant
- Open market operations and QE leave constant the consolidated level of government liabilities

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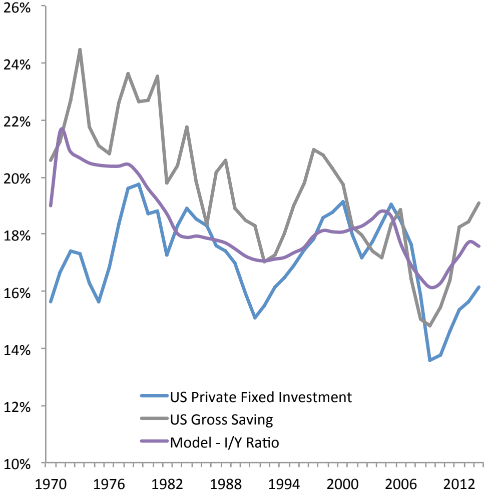
Investment to Output Ratio



K/Y Ratio and Average Product

Capital to output ratio:

- The K/Y ratio in our model rises from 0.92 to 1.22 from 1970 to 2015
- K/Y ratio for business capital (relative to business value added) averages 1.10 to 1.43 from 2000-2015
- K/Y ratio for business capital (relative to gross domestic income) averages 0.83 to 1.08 from 2000-2015

Average product of capital:

- The average product of capital falls from 18.1% in 1970 to 17.5% in 2016
- Measure average product as output less depreciation and payments to labor divided by capital stock: $p_k K$
- APK averages 14.6% from 2000-2015 period measured as business profits divided by business capital

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Markups and Intermediation Wedge

Markups:

$$\frac{K}{Y} = A_k^{\sigma-1} \left(\frac{(1-\mu)\alpha}{r+\delta} \right)^{\sigma}$$

Intermediation Wedge:

$$\frac{K}{Y} = A_k^{\sigma-1} \left(\frac{\alpha}{r+\omega+\delta} \right)^{\sigma}$$

- Rising product market markups could explain falling labor share
- Increase in intermediation wedge would not generate changes in income shares

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