#### Late-in-Life Risks and the Under-Insurance Puzzle

John Ameriks Joseph Briggs Andrew Caplin Vanguard NYU NYU

Matthew D. Shapiro Christopher Tonetti Michigan Stanford GSB

### Long Term Care Expenditure

- Macroeconomic
  - Expenditures on long-term care services in 2004 accounted for 8.5 % of all health care spending in the United States and about 1.2% of GDP (Brown Finkelstein (2011))
- Individual
  - One in three 65-year-olds will eventually enter a care facility (Brown Finkelstein (2011))
  - Private nursing home room averages \$84K per year
- Less than 10% of households own LTC Insurance (LTCI)
  - Even in relatively wealthy population considered today, only 22% own LTCI

### Explanations of Low Insurance Holdings

- People value wealth relatively little in state when need help with activities of daily living (ADLs).
  - Crowding out by public insurance (Pauly (1990))
  - Strong bequest motives (Lockwood (2015))
- 2 Behavioral resistance
  - Framing; Status quo; etc.
  - Annuity puzzle?
- There exists a substantial demand for insurance for ADL state that isn't currently met in the market
  - Underdeveloped market due to product imperfections and failure to insure relevant risk

### LTCI Market

- State of LTCI market
  - Typical policy at age 65 covers only two-thirds of expected present discounted value of LTC expenses (BF (2011)).
  - Loads up to 32 cents per dollar of coverage (BF (2011)).
  - Stallard (2011): "Half of [the elderly] disabled population does not meet the eligibility requirements for tax qualified LTC insurance policies due to not satisfying either HIPAA's ADL trigger definitions or its cognitive impairment trigger."
  - Risk of premium increases to continue coverage
  - Restriction on use of payouts
  - Companies selling "meaningful policies" decreased from 102 to  ${\approx}12$  from 2002-2009 (Cohen et.al (2013))
- Key point: Observed holdings might not reflect desire to insure LTC.

### Question for Today

- Is there room for expansion of the private LTCI market?
  - Can we quantify to what extent proposed explanations contribute the low observed demand?
  - Requires measurement of counterfactual demand.
  - Better understanding of demand is policy relevant.

### Our Approach

- Study demand for hypothetical ADL insurance (ADLI)
  - Asset that pays out in the state where an individual needs help with ADLs
- Measure motives and predict demand
  - Sample: Vanguard Research Initiative
  - Measurement: Estimate individual preferences using SSQs
  - Model: Life-cycle model with incomplete markets and LTC-state dependent utility function

#### The Underinsurance Puzzle

- Result: model predicts significantly higher demand for improved product than observed holdings indicate
- Robust to alternative samples, prices, parameter estimates, etc.
- Reasonable: follows from simple feature of expressed preferences
- Annuities similar (not covered today)

#### Beyond the Model

- Stated Demand
  - Survey demand for product that is identical to modeled
  - Again indicates potential for market expansion
- What do we learn from comparing two quantitative demand measures?
  - Promising indications on missing elements: adverse selection, family motives, and survey response patterns

### Outline

- Model and motives
- Introduce sample
- Strategic Survey Questions (SSQs)
  - Introduce and analyze responses
- Estimation and interpretation of preferences
- Analyze model predicted ADLI demand
  - Detail LTCI puzzle
  - Explore determinants of demand
  - Analyze properties of demand functions
- Introduce and analyze stated demand
- Investigate gap between demand measures

### Related Literatures

- LTC and LTCI Pauly (1990), Cutler (1996), Finkelstein McGarry (2006), Brown Finkelstein (2007, 2008, 2011), Hendren (2013), Koijen Yogo (2015)
- Health State Utility Arrow (1974), Viscusi Evans (1990), Lillard Weiss (1997), Finkelstein, et.al (2009, 2013), Brown et.al (2015)
- Structured Surveys Barsky et.al (1997), Ameriks et.al (2010), Paweenawat Townsend (2012), Beshears, et.al (2013), Ameriks et.al (2015), Brown et.al (2015), Wiswall Zafar (2015), Fuster Zafar (2015), Attanasio, et.al (2015)
- Life-cycle Models and Saving Motives Modigliani (1986), Laibson et.al (1998), Palumbo (1998), Hubbard et.al (2004), De Nardi (2008), De Nardi, French, Jones (2010), Kopecky Koreshkova (2014), Braun, Kopecky, Koreshkova (2014), Koijen, Van Nieuwerburgh, Yogo (2015), Lockwood (2015)

### Model Setup

- Life-cycle saving model (Ameriks et.al (2015))
- State Variables:
  - Age:  $t \in \{55, 56, ..., 110\}$
  - Wealth:  $a \in [0, \infty)$
  - Income Profile:  $y \in \{y_1, y_2, \dots, y_5\}$
  - Health Status:  $s \in \{0, 1, 2, 3\}$
  - Health Cost:  $h \sim H(t, s)$
  - Gender:  $g \in \{m, f\}$
- Choices:
  - Consumption:  $c \in [0, \infty)$
  - Expenditure on LTC:  $e_{ADL} \in [\chi, \infty)$
  - Savings: a'
  - Use of government care:  ${\it G} \in \{0,1\}$



#### Model: Health

- Health transitions conditional on age, health, and gender  $\pi(s'|t,s,g)$ 
  - Good health (s=0)
  - Sick (s=1)
  - Need help with ADLs (s=2)
  - Dead (s=3)
- Estimated from appropriately conditioned HRS sample



#### Model: Health-dependent Utility Functions

• Healthy or Sick (s=0,1)

$$U(c) = \frac{c^{1-\sigma^i}}{1-\sigma^i}$$

• Bequests (s=3)

$$u(b) = \left(\theta_{beq}^{i}\right)^{-\sigma_{i}} \frac{\left(b + \kappa_{beq}^{i}\right)^{1-\sigma'}}{1-\sigma^{i}}$$

Need LTC (s=2)

$$U(e_{ADL}) = \left(\theta_{ADL}^{i}\right)^{-\sigma_{i}} \frac{\left(e_{ADL} + \kappa_{ADL}^{i}\right)^{1-\sigma^{i}}}{1-\sigma^{i}}$$

### The Vanguard Research Initiative (VRI)

- Sample of approximately 9,000 Vanguard clients aged 55+
  - Singles oversampled relevant subsample for today
  - Not representative sample of US: wealthier, more educated, etc.
- So far, 4 surveys
  - Wealth, income, expectations
  - Annuities, LTC, public care, and bequests
  - Samily structure, intervivos transfers, portfolio choice
  - 4 Labor history and retirement expectations
- Needed for this study:
  - Appropriate sample and measurement of state variables
  - Questions to separate saving motives
  - Stated mesaure of same product considered in model
  - Primarily draw from surveys 1 and 2
- Website: http://ebp-projects.isr.umich.edu/VRI/

## The Sample

		Characteristics of the Sample								
		Wealth								
	<u>N</u>	<u>Mean</u>	<u>75p</u>	90p						
Full Sample	1087	745,274	115,000	271,720	543,191	1,012,263	1,587,400			
Employer Only	162	557,026	52,473	168,150	392,926	836,400	1,161,000			
				Demogr	aphics					
	Educ	ation	P	<u>Healt</u>	<u>.h</u>	Sex				
	< College	$\geq$ College	Poor or <u>Fair</u>	Good	Very Good or Excellent	Male	Female			
Full Sample	25.7%	74.3%	5.2%	22.5%	72.2%	44.3 %	55.7%			
Employer Only	37.7%	62.3%	4.3%	29.0%	66.7%	45.1%	54.9%			

### Strategic Survey Questions

- Strategic Survey Questions (SSQs) are designed to provide data on preferences using answers to strongly identifying hypothetical questions
- The structure of SSQs:
  - describe hypothetical environment
  - describe hypothetical state
  - describe hypothetical future
  - describe hypothetical choice set
  - verify understanding
  - record a choice

### The Four SSQs

- We've developed and fielded four types of SSQs, each of which is designed to identify different saving motives:
  - Risk aversion SSQ (BJKS modification)
  - 2 LTC state utility function SSQ
  - Bequest utility function SSQ
  - Public care aversion SSQ
- Will now walk through SSQ 3.

#### SSQ 3 Math Problem

• Allocate wealth between LTC and bequest state

• Translate following optimization problem:

$$\max_{\{x_1, x_2 | x_1 + x_2 = W\}} \quad \frac{\theta_{ADL}^{-\sigma} (x_1 + \kappa_{ADL})^{1-\sigma}}{1 - \sigma} + \frac{\theta_{beq}^{-\sigma} (x_2 + \kappa_{beq})^{1-\sigma}}{1 - \sigma}$$
$$x_1, x_2 \ge 0; \ x_1 \ge -\kappa_{ADL}; \ x_2 \ge -\kappa_{beq}.$$

• *W* = \$100,000

# SSQ 3 (1/4)

Suppose you are 85 years old, live alone, rent your home, and pay all your own bills. You know with certainty that you will live for only 12 more months and that you will need help with \*ADLs for the entire 12 months. You have **\$100,000** that you need to split into Plan E and Plan F.

- Plan E is reserved for your spending. From Plan E, you will need to pay all of your expenses, including long-term care and any other wants, needs, and discretionary purchases.
- Plan F is an irrevocable bequest.

## SSQ 3 (2/4)

Here are the rules for this scenario.

- You have no money other than the \$100,000.
- Other than Plan E, you have no other resources available to help with your long-term care. You have to pay for any long-term care you may need from Plan E.
- Any money in Plan E that you do not spend cannot be given away or left as a bequest.
- You have full insurance that covers all of your hospital, doctor, and medications, but you have no long-term care insurance.
- There is no public-care option or Medicaid if you do not have enough money to pay for a nursing home or other long-term care.

# SSQ 3 (3/4)

Subset of stated comprehension questions:

- In the hypothetical scenario, if you want to buy anything during this year, do you have money aside from what is in Plan E?
  - Yes
  - No
- In the hypothetical scenario, money in Plan F is available
  - Only as a bequest
  - Only for spending in the next 12 months
  - Both as a bequest and to spend in the next twelve month
  - Neither as a bequest nor to spend in the next 12 months

# SSQ 3 (4/4)



	Section 1	Section 2	Section 3	Section 4
			<u>Click here f</u>	or complete scenario
Please make your decision on splitting money into Pla slider to the right. To put more money in Plan F, mov you know how much you will have to spend and how	n E and Plan F by clicki e the slider to the left. much you will leave as	ng on the scale belo The numbers in the a bequest.	w. To put more money in box will change as you m	Plan E, move the ove the slider to let
lease move the slider to see how it works. When you	u are ready, place the s	slider at the split you	want and click NEXT to e	nter your choice.
Plan F				Plan E \$50,000
\$50,000 You will leave the			Yoab	ou will have the ove amount during
above amount as an irrevocable bequest.			th yo Al	e next year when u need help with DLs.
			Previous Ne	ext

#### SSQ Responses

• SSQ 3 response histogram - Allocation to LTC state



SSQ Responses - Credibility

#### Internal credibility

- Correlation between SSQ responses
- Post-survey general reflection questions

#### • External credibility

- Performance on comprehension checks
- Correlation with behaviors and expectations

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#### Preference Estimation

- Preferences in model overidentifed at individual level from SSQ responses
  - 9 SSQs, 6 preference parameters
  - Identification by design
- Estimate heterogeneous preferences
  - Unique parameter set for each individual
- Estimation Methodology
  - Assume additive error on responses
  - Maximum Likelihood Estimation



#### **Estimated Preferences**

Marginal	Distribution	of	Parameters
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	<u>\sigma</u>	$\theta_{ADL}$	KADL	$\theta_{beq}$	К <sub>beq</sub>	ΨG	
10%	2.04	.27	-82.44	.16	3.23	19.97	
25%	3.02	.43	-50.65	.28	11.70	39.75	
50%	4.52	.86	-9.45	.55	125.72	59.99	
75%	6.74	2.26	46.23	2.26	362.64	99.87	
90%	10.11	6.45	148.81	7.72	781.45	178.34	
Median Standard Errors	.13	.38	10.71	.82	18.44	.35	

#### **Estimated Preferences**

 Recall optimization problem posed in SSQ 3 – Allocation between LTC and bequest state

$$\max_{\{x_1, x_2 | x_1 + x_2 = W\}} \quad \frac{\theta_{ADL}^{-\sigma} (x_1 + \kappa_{ADL})^{1-\sigma}}{1-\sigma} + \frac{\theta_{beq}^{-\sigma} (x_2 + \kappa_{beq})^{1-\sigma}}{1-\sigma}$$
$$x_1, x_2 \ge 0; \ x_1 \ge -\kappa_{ADL}; \ x_2 \ge -\kappa_{beq}.$$

Plot percentiles of responses implied by estimated preferences



Share of wealth allocated to LTC state

### Model-implied ADLI Demand

- Use estimated preferences and risks in life-cycle saving model to recover demand for hypothetical LTC insurance
- Activities of daily living insurance (ADLI)
  - is an asset that pays out in state when need help with ADL
  - is priced to be actuarially fair (conditioning on age, gender, health)
  - has no default risk
  - is inflation protected

#### Model-implied ADLI Demand

- Actuarially fair pricing means expect zero profits from sale of insurance product
- Price p(t, s, g), such that spending  $\tilde{y} \times p(t, s, g)$  purchases payout  $\tilde{y}$  per year when need LTC
- Given p and preference vector Θ<sub>i</sub>, demand for ADLI as a function of idiosyncratic states is

$$D^{i}(a, y, t, s, h, g, \Theta) = \arg \max_{\tilde{y}} V^{\Theta}(a - p(t, s, g)\tilde{y}, \hat{y}, t, s, h, g)$$
$$\hat{y} = y + \tilde{y}$$

### LTCI Puzzle

- 63 percent of respondents are predicted to have positive demand for ADLI
- 22 percent of respondents hold private LTCI
- Significant intensive margin demand as well



ADLI Income Demand for Individuals with Positive Demand

#### LTCI Puzzle Across Wealth and Income

Fraction of respondents with private LTCI and predicted to demand ADLI



### LTCI Puzzle - Robustness

- LTCI puzzle robust to:
  - A 3% return on saving
  - A 10% load
  - Assuming multiplicative response errors
  - Using parameters estimated from wealth data alone
  - In a subsample linked to Vanguard through an employers choice
  - When reweighting the sample to match HRS wealthholder statistics

	% > 0	mean	р5	p10	p25	p50	p75	p90	p95
Baseline	63	40,023	0	0	0	19,205	61,546	106,479	155,650
Alt. Estimates									
r=.03	56	35,456	0	0	0	10,245	55,419	100,394	134,718
10% Load	56	36,289	0	0	0	11,513	57,425	103,093	140,493
Mult. Errors	71	43,860	0	0	0	30,977	64,762	107,298	149,660
Wealth Params	87	82,012	0	0	69,058	80,641	101,060	139,767	167,383
Subsamples									
Employer Sample	57	25,776	0	0	0	10,137	41,441	75,483	99,182
HRS weighted	51	22,544	0	0	0	0	31,735	73,331	101,136

#### **ADLI Demand - Characteristics**

Price elasticity and willingness to pay (WTP) (evaluated at predicted demand) for ADLI for respondents predicted to demand positive ADLI



#### **ADLI Demand - Characteristics**

Price elasticity (evaluated at predicted demand) for ADLI for respondents predicted to demand positive ADLI by wealth and income quintiles



#### **ADLI Demand - Characteristics**

WTP (evaluated at predicted demand) for ADLI for respondents predicted to demand positive ADLI by wealth and income quintiles



### ADLI Demand - What motivates purchase?

		ADLI purchas	se decisio	חכ		
	σ	$\theta_{ADL}$	KADL	$\theta_{beq}$	$\kappa_{beq}$	ψG
Purchase	5.66	3.67	-5.85	3.86	168.89	78.30
Don't Purchase	4.32	25.41	40.93	25.97	313.58	81.69
	Age	Income Quint	<u>Wealth</u>	<u>Gender</u>	<u>Health</u>	
Purchase	68.70	3.2	862,408	.45	1.05	
Don't Purchase	66.85	2.9	539,942	.42	1.07	

Average parameters and demographic characteristics stratified by predicted ADLI purchase decision

- Those predicted to purchase/not purchase have similar states, different wealth levels, and different preferences
  - Higher risk aversion, stronger ADL utility, lower bequest luxury

### Directly Elicited ADLI Demand

- Want: model free measure
- Use survey to recover stated demand for identical product
- Describe and confirm understanding of hypothetical LTC insurance

### Survey Description of ADLI

Please suppose that you are offered a hypothetical new form of insurance called **\*ADL insurance** with the following features:

- You pay a one-time, nonrefundable lump sum to purchase this insurance.
- If you need help with activities of daily living (\*ADLs), you will immediately receive a monthly cash benefit indexed for inflation.
- For each **\$10,000** you pay for this insurance, you will receive **\$Y** per month indexed for inflation in any month in which you need help with **\***ADLs
- The monthly cash benefit is set at the time of purchase and is not dependent on your actual expenses.
- There is **no restriction** on the use of the insurance benefits. You are free to use benefits in any way you wish: to pay for a nursing home; a nurse to help at home; for some other form of help; or in literally any other way you would like.
- An impartial third party who you trust will verify whether or not you need help with \*ADLs immediately, impartially, and with complete accuracy.
- The insurance is priced fairly based on your gender, age, and current health.
- There is no risk that the insurance company will default or change the terms of the policy.

### Stated ADLI Demand

- Higher fraction of state positive demand (29%) than observed holdings suggest
- Poor existing LTCI products partially, not fully, resolve LTCI puzzle



Figure: Fraction of Population Owning LTCI: This figure presents various measures of the fraction of the population with positive LTCI ownership. Column 1 is actual holdings of a private LTCI in the sample. Column 2 is stated ADLI demand. Column 3 is the union of private ownership and stated demand. Column 4 is model predicted ADLI demand.

Model-implied vs. Stated ADLI Demand: Intensive Margin

- Sizable intensive margin stated demand for many people
- Higher predicted demand than stated demand

	mean	р5	p10	p25	p50	p75	p90	p95	_
Stated	6,451	0	0	0	0	6,000	18,000	36,000	
Modeled	40,023	0	0	0	19,205	61,546	106,479	155,650	
Modeled-Stated	33,727	-18,382	-7,200	0	12,947	56,920	100,424	148,472	

### A Model Misspecification Guide

- Why does model have such higher demand than observed holdings
- Use difference in demand estimates to check for misspecification
  - Develop a method to detect systematic patterns:

• 
$$D_i - S_i = G(x_i, \Theta_i, q_i)$$

• 
$$G(x_i, \Theta_i, q_i) \approx g_x(x_i) + g_{\Theta}(\Theta_i) + g_q(q_i)$$

- $g_X$ ,  $g_{\Theta}$  non-parametrically approximated
- $g_q$  linear specified as an indicator variable

• 
$$D_i - S_i = \beta^x C_i^x + \beta^{\Theta} C_i^{\Theta} + \Gamma q_i + \epsilon_i$$

#### Predictors of the Demand Gap

Regressing demand difference (modeled-surveyed) on indicator of either										
	above median or holder of indicated characteristic									
			A	<b>DLI</b> differ	ence					
	1	2	<u>3</u>	4	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>		
II Transfers	8,638*							9,703*		
	(5,792)							(6,509)		
$\mathbb{I}_{child}$		5,669						3,204		
		(6,049)						(7,139)		
<b>I</b> <sub>Real</sub> Estate			-2,424					-1,446		
			(6,147)					(6,113)		
II College				-4,482				-2439		
0				(5,979)				(6,152)		
II <sub>Comp. Test</sub>					-7,208*			-8,037*		
					(5,272)			(5,334)		
II <sub>Family</sub> Care						-71		-2,614		
5						(5,442)		(5,791)		
I <sub>ADL help</sub>							-8,568**	-8,805**		
							(5,175)	(5,163)		

### Conclusion

- LTCI is a large risk that few people insure
- In VRI find high predicted demand for ideal products
  - Predicts market expansion if products improve
  - Product highly valued and demand is inelastic
  - Heterogeneity in demand largely driven by heterogeneity in preferences
- Stated demand also indicates higher demand
  - Market potential not as large as model predicts
- Hints as to next steps
- Independent measures of preferences valuable, method general

## Credibility of SSQ Responses: Specific Comprehension Questions

	SSQ 1	SSQ 2	SSQ 3	SSQ 4
Number of questions	6	9	3	2
All correct, 1 <sup>st</sup> try	46.3%	18.6%	55.4%	77.3%
All correct, 2 <sup>nd</sup> try	75.1%	55.5%	81.9%	94.1%
$\leq 1$ wrong, $2^{nd}$ try	93.4%	80.8%	96.2%	99.5%

## Credibility of SSQ Responses: General Comprehension Questions

Overall, how clear were the tradeoffs that the hypothetical scenarios asked you to consider? Overall, how well were you able to place yourself in the hypothetical scenarios and answer these questions? How much thought had you given to the issues that the hypothetical scenarios highlighted before taking the survey?

Response	Percent
Very Clear	51.8
Somewhat Clear	39.7
Somewhat Unclear	7.4
Very Unclear	1.1

 And answer these questions?

 Response
 Percent

 Very Well
 23.1

 Moderately Well
 60.5

 Not very well
 14.2

 Not very well at all
 2.2

-		
	Response	Percent
	A lot of thought	29.5
	A little thought	52.1
	No thought	18.4



## Credibility of SSQ Responses: Internal Coherence

	Correlation of SSQ Responses within Individuals								
	SSQ 1a	SSQ 1b	SSQ 2a	SSQ 2b	SSQ 2c	SSQ 3a	SSQ 3b	SSQ 3c	SSQ 4a
SSQ 1a	1.00								
SSQ 1b	0.44	1.00							
SSQ 2a	-0.01	0.04	1.00						
SSQ 2b	-0.04	-0.01	0.61	1.00					
SSQ 2c	-0.08	0.07	0.55	0.56	1.00				
SSQ 3a	-0.01	-0.08	-0.11	-0.04	-0.11	1.00			
SSQ 3b	-0.06	-0.08	0.04	0.04	0.023	0.78	1.00		
SSQ 3c	-0.08	-0.08	0.07	0.08	0.07	0.63	0.86	1.00	
SSQ 4a	-0.03	-0.00	0.04	0.06	0.04	-0.11	-0.10	-0.08	1.00

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## Credibility of SSQ Responses: External Validation

	SSQ 3a	SSQ 3b	SSQ 3c
Average ADL Cost	.03	.05**	.07**
	(.02)	(.02)	(.03)
Prob. Family Cares	-56.33	-90.82*	-135.51**
for ADLs	(40.86)	(47.95)	(60.41)
Above Median	-4,858.13**	-9,401.06***	-11,331.22***
Transfers	(2,307.98)	2,697.35)	(3,391.45)
<b>Opinion of Public</b>	-2,423.81*	80.29	1,466.69
ADL Facility	(1,358.17)	(1,586.00)	(1,991.96)

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### ADL Risk is Prevalent and Sizable

Probability of needing help with ADLS for x years



- Males: 55%  $\geq$ 1 year, 25%  $\geq$ 3 years, 13%  $\geq$ 5 years
- $\bullet$  Females: 65%  ${\geq}1$  year, 40%  ${\geq}3$  years, 24%  ${\geq}5$  years

#### Individual's Problem

$$\begin{split} V(a, y, t, s, h, g) &= \max_{a', c, e_{ADL}, G} \mathbb{I}_{s \neq 3} (1 - G) \left\{ U_s(c, e_{ADL}) + \beta E[V(a', y, t + 1, s', h')] \right\} \\ &+ \mathbb{I}_{s \neq 3} G \left\{ U_s(\omega_G, \psi_G) + \beta E[V(0, y, t + 1, s', h')] \right\} + \mathbb{I}_{s = 3}\{v(b)\} \\ \text{s.t.} \\ a' &= (1 - G)[(1 + r)a + y(t) - c - e_{ADL} - h] \ge 0 \\ e_{ADL} \ge \chi \text{ if } (G = 0 \land s = 2) \\ e_{ADL} &= \psi_G \text{ if } (G = 1 \land s = 2) \\ c &= \omega_G \text{ if } (G = 1 \land (s = 0 \lor s = 1)) \\ b &= \max\{(1 + r)a - h', 0\} \\ U_s(c, e_{ADL}) &= \mathbb{I}_{s \in \{0,1\}} \frac{c^{1 - \sigma}}{1 - \sigma} + \mathbb{I}_{s = 2} \theta_{ADL} \frac{(e_{ADL} + \kappa_{ADL})^{1 - \sigma}}{1 - \sigma} \\ v(b) &= \theta_{beq} \frac{(b + \kappa_{beq})^{1 - \sigma}}{1 - \sigma} \end{split}$$

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#### **Estimating Preferences**

- Let the response of the  $k^{th}$  SSQ by individual i be  $\hat{z}_k(\Theta_i)$
- Observed responses as true response  $z_k(\Theta_i)$  plus measurement error
- $z_k(\Theta_i)$  comes from FOC of math SSQs

$$\hat{z}_{k}(\Theta_{i}) = z_{k}(\Theta_{i}) + \hat{\epsilon}_{k,i}$$
• Let  $\epsilon_{k,i} \sim \mathbb{N}(0, \sigma_{k,i}^{2})$  and  $\sigma_{k,i} = \underbrace{\bar{\sigma}_{i}}_{\text{individual component}} \times \underbrace{W_{k}}_{\text{SSQ k component}}$ 

$$\mathcal{L}_{k}(\Theta_{i}, \bar{\sigma}_{i} | \hat{z}_{k,i}) = \begin{cases} F_{\sigma_{k,i}^{2}}(-z_{k}(\Theta_{i})) & \text{if } \hat{z}_{k,i} = 0\\ f_{\sigma_{k,i}^{2}}(\hat{z}_{k,i} - z_{k}(\Theta_{i})) & \text{if } 0 < \hat{z}_{k,i} < W_{k}\\ 1 - F_{\sigma_{k,i}^{2}}(W_{k} - z_{k}(\Theta_{i})) & \text{if } \hat{z}_{k,i} = W_{k}. \end{cases}$$

and

$$\mathcal{L}(\Theta_i, \bar{\sigma}_i | \hat{Z}_i) = \prod_{k=1}^{9} \mathcal{L}_k(\Theta_i, \bar{\sigma}_i | \hat{z}_{k,i}).$$

