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Nursing Home Choice, Family Bargaining and Optimal Policy in a Hotelling Economy

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Abstract

The family plays a central role in decisions relative to the provision of long term care (LTC). We develop a model of family bargaining to study the impact of the distribution of bargaining power within the family on the choices of nursing homes, and on the location and prices chosen by nursing homes in a Hotelling economy. We show that, if the dependent parent only cares about the distance, whereas his child cares also about the price, the mark up rate of nursing homes is increasing in the bargaining power of the dependent parent. We contrast the laissez-faire with the social optimum, and we show how the social optimum can be decentralized in a first-best setting and in a second-best setting (i.e. when the government cannot force location). Finally, we explore the robustness of our results to considering families with more than one child, and to introducing a wealth accumulation motive within a dynamic OLG model, which allows us to study the joint dynamics of wealth and nursing home prices. We show that a higher capital stock raises the price of nursing homes through higher mark up rates.

Keywords: Family bargaining, long term care, nursing homes, spacial competitition, optimal policy, OLG models.

JEL codes: D10, I11, I18.

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1 Introduction

Due to the ageing process, the provision of long-term care (LTC) to the dependent elderly has become a major challenge for advanced economies. According to the EU (2015), the number of dependent persons in the Euro Area is expected to grow from about 27 million persons in 2013 to about 35 million persons by the year 2060. Although that forecast depends on the underlying scenarios regarding future mortality and disability trends, it is nonetheless widely acknowledged that, whatever the postulated scenarios are, in any case there will be a substantial rise in LTC needs in the next decades.

The provision and funding of LTC are usually carried out by three distinct economic institutions: the family, the market and the State. Among these, the most active institution remains, undoubtedly, the family. As emphasized by Norton (2000), about two thirds of the supply of LTC is provided informally by the family (spouses and children), whereas the remaining consists of formal care, either at home or in nursing homes. As far as the funding of LTC is concerned, the literature has emphasized that the market for private LTC insurance is underdeveloped. This is the well-known LTC private insurance puzzle (see Brown and Finkelstein (2011). Despite a large probability (between 35 % and 50 %) to enter a nursing home at some point in one's life (see Brown and Finkelstein 2009), and despite large costs of being in a nursing home, only a limited fraction of the population at risk purchases a private LTC insurance.[1] Finally, although the Welfare State has recently evolved to provide some protection against LTC risks (e.g. in Germany), in most countries public intervention remains limited in comparison with the large costs induced by LTC (see Cremer et al. 2012).

The central role played by the family in LTC provision raises complex issues from an economic perspective. The family is a *collective* agent, which is composed of various individuals (the dependent, his spouse, his children), who pursue different goals, and face different time and budget constraints. Hence, when studying how LTC needs are satisfied, a particular attention must be paid to the modelling of the collective decision process at work within the family. Two classes of models were developed to study LTC decisions (e.g. choice of living arrangements). On the one hand, models of *non-cooperative* decision-making, where family members play Nash.[2] Those models showed that, when the health of the dependent is a public good in the family, coordination failures arise, leading to suboptimal outcomes. On the other hand, the literature also includes models of *cooperative* decision-making, where the selected option maximizes the well-being of the family defined as a weighted sum of the utilities of its members, the weights reflecting the bargaining power of each family member.[3]

Models of family bargaining point to an important, but often neglected,

[1]According to Brown et al. (2007), only about 9 to 10 % of the population at risk has purchased a private LTC insurance in the U.S.

[2]Models of that kind include Hiedemann and Stern (1999), Stern and Engers (2002), Konrad et al (2002), Kureishi and Wakabayashi (2007), and Pezzin et al. (2009).

[3]Models of that kind include, among others, Hoerger et al. (1996), Sloan et al. (1997) and Pezzin et al. (2007).

determinant of social outcomes: the *distribution of bargaining power* within the family.[4] As stressed by Sloan et al (1997), the dependent parent and his children can disagree regarding the kind of supply (formal or informal) of LTC, because they do not have the same preferences. Hence, the option that will emerge depends, at the end of the day, on the distribution of bargaining power within the family. Sloan et al (1997) stressed that the bargaining power of the parent depends on three main features: first, his degree of cognitive awareness (which could limit his capacity to take part to the decision); second, his number of children (which can allow the dependent parent to make his children compete for gifts); third, his wealth (the strategic bequest motive).

The goal of this paper is to explore further the consequences of the distribution of bargaining power within the family on LTC outcomes, by considering its impact on the *prices* and *location* of nursing homes. The underlying intuition is that the distribution of bargaining power in the family may not only affect whether the dependent parent is sent or not to a nursing home, but, also, where the nursing homes are located and which prices they charge for LTC provision.

The reason why we focus on the location and prices of nursing homes is that those two dimensions drive the choice of a particular nursing home. According to the recent study by Schmitz and Stroka (2014), the probability of choosing a nursing home decreases in distance and price. Hence, the lower the nursing home price and the closer the location of the nursing home to the previous household, the more likely are the elderly to choose this nursing home.[5] Those two dimensions - distance and price - are the most important determinants of nursing home choices, and matters more than nursing home (reported) quality, which was found to have no significant effect on the choice of nursing homes.

In order to study the impact of the distribution of bargaining power within the family on the location and prices of nursing homes, we develop a model of family bargaining where a family, composed of a dependent parent and a child, must choose between two nursing homes, which are located along a line, in the spirit of Hotelling's canonical model (Hotelling 1929). In the baseline version of the model, we consider a parent who is interested in minimizing the distance between the nursing home and the location of his child (to have more visits), without concern for the price, whereas the child, although caring also about the distance, wants to avoid too large LTC expenditure. Within that baseline model, we also examine the design of an optimal public policy, under different sets of available instruments (i.e. constraining the location of nursing homes or not). In a second stage, we explore some extensions of this baseline model, to discuss the robustness of our results to relaxing some assumptions. First, we consider the case where the dependent has several children. Second, we develop a dynamic overlapping generations model (OLG) to examine how

[4] On the impact of the distribution of bargaining power on time allocation, see Konrad and Lommerud (2000). de la Croix and Vander Donkt (2010) and Leker and Ponthiere (2015) studied the impact of bargaining on education outcomes.

[5] Schmitz and Stroka (2014) show that about 52 percent of individuals were admitted to nursing homes within 10 minutes travel time to their previous households, and that the average distance to the chosen nursing home is 9.58 minutes travel time.

the distribution of bargaining power within the family affects the accumulation of wealth and the dynamics of nursing home prices over time.

Anticipating on our results, we find, at the laissez-faire, that the principle of maximum differentiation holds: nursing homes locate at the far extreme of the Hotelling line. The mark up rates and thus, prices applied by nursing homes depend on how the bargaining power is distributed within the family. The higher the bargaining power of the dependent elderly is, and the higher the mark up rate is. If, on the contrary, the degree of cognitive awareness of the dependent is so low that the child has full power, then it can be the case, when the child only cares about the price (and not about the distance), that the mark-up rate reduces to zero. Thus the degree of mark up in the nursing sector depends strongly on how the bargaining power is distributed within the family. That laissez-faire situation is contrasted with the utilitarian social optimum, where nursing homes should locate in the middle of each half of the line and prices should be set to marginal cost. This optimum can be decentralized in two ways. First, the government could force nursing-home locations and subsidize them to achieve pricing at the marginal cost. If the government cannot force location, it needs in addition a non linear subsidy on location, which is the same across facilities. This taxation scheme depends on the distribution of bargaining power and on the preferences for the distance of parents and children.

Regarding the robustness of our results, we show that our main results concerning the laissez-faire and the social optimum carry on qualitatively when we increase the number of children of the dependent parent. Note, however, that, within the OLG model with wealth accumulation, it could be the case, if the motive for transmitting wealth to the children is sufficiently strong, that the mark up rate is decreasing - and not increasing - with the bargaining power of the dependent parent. Moreover, the mark up rate is here decreasing with the interest rate, since a higher interest rate raises the opportunity cost of LTC expenditures, by preventing further wealth accumulation. Thus a higher capital stock raises the price of nursing homes through higher mark up rates. Our analysis of the joint dynamics of capital accumulation and nursing home prices reveals also that there exists, under mild conditions, multiple stationary equilibria (some being unstable), with a positive correlation between wealth and the price of nursing homes.

Our paper is related to several aspects of the literature on LTC. First, it is related to models of family bargaining, such as Hoerger et al (1996) and Sloan et al (1997), which studied how family bargaining affects the choice of formal versus informal LTC provision, as well as the choice of living arrangement. Our contribution with respect to those papers is to study how nursing homes react strategically to the structure of the family, and set prices according to the distribution of bargaining power between the dependent and his child. Our paper is also related to the literature on location games in the context of LTC, such as Konrad et al (2002) and Kureishi et Wakabayashi (2007). While those papers studied the strategic location of children with respect to a given nursing home location, we do the opposite, and study the strategic location of nursing homes with respect to a given location of children. We also complement papers in industrial organization applying Hotelling's model to health issues, such as Brekke et al (2014), who considered the competition in prices and quality among hospitals. Our paper complements this IO approach by considering interactions between family bargaining, prices and location outcomes.[6] Finally, we also complement the literature on optimal policy in the context of LTC, such as Jousten et al (2005) and Pestieau and Sato (2008). We complement those papers by considering the impact of family bargaining on LTC outcomes in terms of prices and location of nursing homes, and by exploring the adequate public intervention when nursing homes behave strategically.

The rest of the paper is organized as follows. Section 2 presents the main assumptions of the model. Section 3 characterizes the laissez-faire, and explores the links between the distribution of bargaining power in the family and the mark up rate of nursing homes. The social optimum and its decentralization are studied in Section 4. Section 5 extends our economy to several children per dependent. Section 6 considers a dynamic OLG variant of our baseline model. Conclusions are drawn in Section 7.

2 The model

There exists a continuum of families composed of a child and of a dependent parent.[7] Families are uniformly distributed on the line $[0, L]$, so that density at each point on the line is $1/L$.

Each dependent parent needs to enter a nursing home.[8] There exist two nursing homes in the economy, which are denoted by $\{A, B\}$, and located on the same $[0, L]$ line as families.

2.1 Preferences: children

The child derives utility from his consumption, which is equal to his exogenous income *minus* the price of the nursing home where his parent is located.[9] The child also derives some disutility from being far from the nursing home where his parent is located. The intuition is that visiting the parent at the nursing

[6] The interactions between bargaining and spatial competition were also studied by Bester (1989), who considered a Hotelling model where prices are the outcome of bargaining between consumers and firms. In our model, the bargaining occurs in families, that is on the consumer side only.

[7] For simplicity, we suppose here a unique degree of dependence, which is shared by all parents with certainty.

[8] We consider only families for whom there is no other option (such as staying at the child's place) than to enter a nursing home, either because of the severity of the dependency or because it entails too high disutility to the child. This is equivalent to assuming that a previous (unmodeled) decision step where families decide whether to take care of their dependant elderly or to send him to a nursing home, already took place. From this previous decision step, we focus only on those families who decide to send their parent to a nursing home.

[9] Section 6 considers a dynamic economy where the income is not exogenous anymore, but depends on wages and wealth received from the parent.

home is costly in terms of time, and that this time cost is increasing with the distance between the child and the nursing home of the parent.

For the sake of analytical tractability, the utility of a young agent is assumed to be quasi-linear:

$$U_c = w - p_i - \gamma x_i^2 \tag{1}$$

where $w > 0$ is the child's income, p_i is the price paid for the nursing home $i \in \{A, B\}$, $\gamma > 0$ represents the intensity of the disutility created by distance between the nursing home and the child's location. The variable $x_i > 0$ is the distance between the child's location and the elderly's nursing home $i \in \{A, B\}$. The quadratic form is standard since Hotelling's (1929) pioneer work.

Note that the child's interests for the distance can be interpreted either as a purely self-oriented concern for being able to visit the parent more often (thanks to a lower distance), or, alternatively, as a form of altruistic concern taking into account the fact that the dependent parent does not want to be too far away from his child (see below). Under the latter interpretation, the parameter γ would capture the extent to which the child is altruistic toward his parent.

2.2 Preferences: dependent parents

The utility of the dependent parent is assumed to depend only on the distance between the nursing home and his child (i.e. the initial location of the family):[10]

$$U_d = -\delta x_i^2 \tag{2}$$

where $\delta > 0$ is the intensity of the parent's disutility created by distance between the nursing home and the child's location.

The intuition behind that formulation is the following. Dependent persons have a limited ability to enjoy consumption, but they care about keeping a link with their family. But since the number of visits depends on traveling costs, and, thus, on the distance between the dependent and his visitor, it is reasonable to suppose that a shorter distance between the child and the nursing home will raise the number of visits, and, hence, the welfare of the dependent parent.

Finally, note that the above formulation presupposes that the dependent parent exhibits no altruism towards his child. Section 6 below will relax that assumption, and consider a (more complete) model where parents care about the wealth they transmit to their children (which is decreasing in the price of nursing home).

2.3 Preferences: the family

Within each family, the parent and the child have quite different interests, and there is *a priori* no obvious reason why they should agree on the choice of a

[10] Note that the utility of the dependent parent does not depend on the price of the nursing home. The reason is that, as shown above, the price of the nursing home is here supposed to be entirely supported by the child.

nursing home. Throughout this paper, we propose to represent the choice of a nursing home as the outcome of a family bargaining process.

The utility of the family is given by the following joint utility:

$$U_f = \theta U_c + (1-\theta)U_d \tag{3}$$

where $\theta \in [0,1]$ represents the bargaining power of the child within the family, whereas $1-\theta$ denotes the bargaining power of the dependent parent.

The distribution of bargaining power can take *a priori* various forms. The case where $\theta = 1$ arises when the child is the only one who takes part to the decision of choosing a nursing home. On the contrary, when $\theta = 0$, it is the parent who selects the nursing home, and the child obeys to what his parent decides. This case may seem a bit extreme, especially in a context where the child is the one who pays for the nursing home. Note, however, that the extent to which this case may arise depends on the prevailing culture within families. Some societies strongly value obedience to the parent, and from that perspective the case where $\theta = 0$ cannot be excluded.

As usual, the parameter θ can be interpreted in different ways. As already mentioned, it may reflect the values to which individuals adhere in a society, concerning the extent to which obeying one's parents is regarded as essential or not. But in the context of LTC, it is also possible that θ reflects, to some extent, the degree of cognitive awareness or ability of the dependent parent. Indeed, if the parent is in a situation of weak dependence, he will definitively have a word to say in the choice of a nursing home. However, if the parent is in a strong state of dependence (i.e. extremely limited autonomy such as an advanced Alzheimer condition), this may reduce the bargaining power of the parent. In that case, it is plausible that the child will choose the nursing home alone ($\theta \to 1$).

When the family opts for the nursing home $i \in \{A, B\}$, its utility is:

$$U_{f,i} = \theta\,(w - p_i) - (\theta\gamma + (1-\theta)\,\delta)x_i^2 \tag{4}$$

3 Laissez-faire

Let us describe the timing of the model. First, nursing homes A and B choose simultaneously their location, a and $L - b$ respectively, on the line $[0, L]$. Second, they simultaneously fix the prices they charge (respectively p_A and p_B), anticipating families' demand and taking the price proposed by the other facility as given. Finally, families choose which nursing home $i \in \{A, B\}$ to send the dependent elderly, taking prices and location as given. This last step determines the demand for each nursing home. As usual in these types of models, we solve it backwards, starting from the families' decisions.

3.1 Families decision

We solve the demand for each nursing home by first identifying the median family, who, by definition, is indifferent between the two nursing homes.

For that family, denoted by m, we have:

$$\begin{aligned} U_{m,A} &= U_{m,B} \\ \theta\,(w-p_A)-(\gamma\theta+(1-\theta)\,\delta)x_{m,A}^2 &= \theta\,(w-p_B)-(\gamma\theta+(1-\theta)\,\delta)x_{m,B}^2 \end{aligned} \tag{5}$$

where $x_{m,A}=m-a$ is the distance between the nursing home A and the median family, while $x_{m,B}=L-b-m$ is the distance between the nursing home B and the median family, with a (resp. $L-b$) the point on the line at which A (resp. B) is located.

Together with the constraint on distances:

$$x_{m,A}+x_{m,B}=L-a-b, \tag{6}$$

we obtain, after some simplifications:

$$x_{m,A}=\frac{\theta(p_B-p_A)}{2(L-a-b)(\gamma\theta+(1-\theta)\,\delta)}+\frac{(L-a-b)}{2} \tag{7}$$

It is straightforward to show that an increase in price p_A decreases $x_{m,A}$. Equivalently, the median is further to the left which means that the demand for nursing home A relative to nursing home B decreases. To the contrary, an increase in p_B increases $x_{m,A}$, meaning that the median is pushed further to the right and thus that the demand for nursing home A relative to nursing home B increases. Moreover, we have:

$$x_{m,B}=\frac{\theta(p_A-p_B)}{2(L-a-b)(\gamma\theta+(1-\theta)\,\delta)}+\frac{(L-a-b)}{2} \tag{8}$$

The comparative statics of $x_{m,B}$ with respect to p_A and p_B are symmetric to those for $x_{m,A}$.

Note that $x_{m,A}$ and $x_{m,B}$ needs to be positive, which is always verified when the difference in nursing home prices satisfy the following condition:

$$-\frac{\gamma\theta+(1-\theta)\,\delta}{\theta}(L-a-b)^2<(p_A-p_B)<\frac{\gamma\theta+(1-\theta)\,\delta}{\theta}(L-a-b)^2. \tag{9}$$

We check ex post that this is effectively the case (see Proposition 1).

Given that all families located on the left of the median family prefer the nursing home A over the nursing home B, the total demand for nursing home A, denoted by $D_A\,(p_A,p_B)$ is equal to $a+x_{m,A}$ when families are uniformly distributed. This is thus equal to

$$\begin{aligned} D_A\,(p_A,p_B) &= a+\frac{\theta(p_B-p_A)}{2(L-a-b)(\gamma\theta+(1-\theta)\,\delta)}+\frac{(L-a-b)}{2} \\ &= \frac{\theta(p_B-p_A)}{2(L-a-b)(\gamma\theta+(1-\theta)\,\delta)}+\frac{(L+a-b)}{2} \end{aligned} \tag{10}$$

Similarly, the total demand for nursing home B, denoted by $D_B(p_A, p_B) = b + x_{m,B}$, is:[11]

$$\begin{aligned} D_B(p_A, p_B) &= b + \frac{\theta(p_A - p_B)}{2(L-a-b)(\gamma\theta + (1-\theta)\delta)} + \frac{(L-a-b)}{2} \\ &= \frac{\theta(p_A - p_B)}{2(L-a-b)(\gamma\theta + (1-\theta)\delta)} + \frac{(L-a+b)}{2} \end{aligned} \tag{11}$$

3.2 Nursing homes' decisions

We now derive nursing homes' decisions.

3.2.1 Setting prices.

Nursing homes choose their price given their locations a and $L-b$ and while taking the price of the other nursing home as given. For the sake of simplicity, we suppose that nursing homes have the same linear cost structure so that the average cost by patient is equal to its marginal cost, $c > 0$.

Nursing home A's profit maximization problem can be written as:

$$\max_{p_A} (p_A - c) D_A(p_A, p_B)$$

Clearly the nursing home market is not competitive, so that each nursing home takes into account that an increase in its price decreases its demand. The first-order condition (FOC) for optimal price is:

$$a + x_{m,A} + \frac{\partial x_{m,A}}{\partial p_A}(p_A - c) = 0 \tag{12}$$

where the first term accounts for the marginal effect of increasing the price on the existing demand on the profit and the second effect represents the marginal decrease in demand due to the increase in price on the profit. This latter effect only concerns the agents to the right of nursing home A and to the left of the median household, that is those who are close enough to nursing home B that they may now change for nursing home B following an increase in the price p_A. An increase in the price will therefore push the median to the left, inducing more agents to use nursing home B.

Similarly, the profit maximization of nursing home B writes:

$$\max_{p_B} (p_B - c) D_B(p_A, p_B)$$

which yields the following FOC:

$$b + x_{m,B} + \frac{\partial x_{m,B}}{\partial p_B}(p_B - c) = 0 \tag{13}$$

[11] Under condition (9), both $D_A(p_A, p_B)$ and $D_B(p_A, p_B)$ are positive.

Solving simultaneously this system of 2 equations (12) and (13), and 2 unknowns, we obtain the following prices:[12]

$$
\begin{aligned}
p_A &= -(\gamma\theta + (1-\theta)\,\delta)\frac{(a+b-L)(a-b+3L)}{3\theta} + c \\
p_B &= -(\gamma\theta + (1-\theta)\,\delta)\frac{(a+b-L)(-a+b+3L)}{3\theta} + c
\end{aligned}
$$

Replacing p_A and p_B in $x_{m,A}$ and $x_{m,B}$, we obtain that these are independent from the cost parameter c and equal to:

$$x_{m,A} = \frac{1}{6}\left(-5a - b + 3L\right) \tag{14}$$

and

$$x_{m,B} = \frac{1}{6}\left(-5b - a + 3L\right) \tag{15}$$

Note that, in equilibrium, these are independent of bargaining powers, meaning that they do not influence the choice of nursing homes location.[13]

3.2.2 Choosing location

The equilibrium location for nursing homes A and B is obtained by selecting the levels of, respectively, a and b that maximize their own profits, taking into account that both prices and demand depend on a and b and while taking the location of the other nursing home as given.

The nursing home A chooses location a maximizing its profit:

$$\max_a \pi_A = \left[-(\gamma\theta + (1-\theta)\,\delta)\frac{a^2 + 2aL + 4bL - b^2}{3\theta} + \frac{c\theta + L^2(\gamma\theta + (1-\theta)\,\delta)}{\theta} - c\right]\left[\frac{1}{6}\left(a - b + 3L\right)\right]$$

After simplifications, the problem of nursing home A can be rewritten as:

$$\max_a \pi_A = \left[-(\gamma\theta + (1-\theta)\,\delta)\frac{\left[a^2 + 2aL + 4bL - b^2 - 3L^2\right]}{3\theta}\right]\left[\frac{1}{6}\left(a - b + 3L\right)\right]$$

Differentiating with respect to a yields after some simplifications:

$$\frac{\partial \pi_A}{\partial a} = \frac{-(\gamma\theta + (1-\theta)\,\delta)}{18\theta}\left[3a^2 + 10aL + 3L^2 + 2b\left(L - a\right) - b^2\right] < 0 \tag{16}$$

The expression inside brackets is positive so that the profits of nursing home A are strictly decreasing with location a. Hence, at the laissez faire, the location for nursing home A is at the left extremity of the segment $[0, L]$: $a^{LF} = 0$.

[12] Note that the usual way of solving this problem consists in solving first the problem faced by nursing home A anticipating the price p_B and second, the problem face by nursing home B anticipating the price p_A. Since in equilibrium, anticipations are realised, solving the above system of equations is equivalent here.

[13] Given that $L - a - b > 0$, one sufficient condition for the above expressions to be positive is that $L \geq 2a$ and that $L \geq 2b$, which is always verified.

The nursing home B chooses location b maximizing its profit :

$$\max_b \pi_B = \left[-(\gamma\theta + (1-\theta)\,\delta)\frac{b^2 + 2bL + 4aL - a^2}{3\theta} + \frac{(c\theta + L^2\delta(1-\theta))}{\theta} - c \right] \left[\frac{1}{6}\,(b - a + 3L) \right]$$

The nursing home B chooses the location b maximizing:

$$\max_b \pi_B = \left[-(\gamma\theta + (1-\theta)\,\delta)\frac{\left[b^2 + 2bL + 4aL - a^2 - 3L^2\right]}{3\theta} \right] \left[\frac{1}{6}\,(b - a + 3L) \right]$$

Differentiating with respect to b yields:

$$\frac{-(\gamma\theta + (1-\theta)\,\delta)}{18\theta}\left[3b^2 + 10bL + 2a\,(L - b) + 3L^2 - a^2\right] < 0 \tag{17}$$

The profits of nursing home B are strictly decreasing with location b. Hence, at the laissez faire, the location for nursing home B is at the right extremity (L) of the segment $[0, L]$: $b^{LF} = 0$.

Hence the principle of the maximum differentiation still holds, and each facility equally shares the market.

Proposition 1 *At the laissez-faire, the two nursing homes A and B locate at the far extremes of the line $[0, L]$:*

$$a^{LF} = b^{LF} = 0$$

Prices in the two nursing homes are equal to:

$$p_A^{LF} = p_B^{LF} = c + \frac{(\gamma\theta + (1-\theta)\,\delta)}{\theta}L^2$$

and the demands are $D_A^{LF} = D_B^{LF} = L/2$.

Proof. $a^{LF} = b^{LF} = 0$ have been replaced in the equations for prices. In equilibrium, $D_A^{LF} = x_{m,A}$ and $D_B^{LF} = x_{m,B}$ defined by (14) and (15) respectively. Note that with this system of prices, (9) trivially holds. ■

As stated in Proposition 1, the two nursing homes share the demand equally, but choose prices above the marginal cost, which is a direct consequence of imperfect competition in the nursing home sector and of maximum differentiation. Interestingly the mark up,

$$MarkUp = \frac{\gamma\theta + (1-\theta)\,\delta}{\theta}L^2$$

depends on both the bargaining power of the child, on the intensity of the young and the old' preferences for the distance between the child's home and the nursing home as well as on the size of the country, represented by L. Regarding the last two determinants, the intuition is straightforward. If agents prefer to be closer to each other (i.e. the intensity of the disutility from being far from each other increase, $\gamma, \theta > 0$ increases), the mark up level increases *ceteris paribus.* Moreover, the mark up is higher the higher L is, which in our set up would mean that the mark up would be higher in larger countries than in smaller ones.

Let us now study the comparative statics of the mark up with respect to θ. As stated in Corollary 1, the mark-up decreases with the bargaining power of the children. Thus, the precise way in which the bargaining power is distributed among the family affects the extent to which nursing homes can have a more or less high mark up, in the sense that a higher bargaining power for the dependent parent will imply a higher mark up for nursing homes.

Corollary 1 *The mark up of nursing homes A and B is decreasing with the bargaining power of the child:*

$$\frac{dMarkUp}{d\theta} = -\frac{\delta}{\theta^2}L^2.$$

Proof. The corollary follows from taking the derivative of $\frac{(\gamma\theta+(1-\theta)\delta)}{\theta}L^2$ with respect to θ. ■

The intuition behind Corollary 1 is straightforward: in our model, only the child cares about prices. Hence, the mark up that nursing homes can charge is limited by the child's willingness to pay for it. If, for instance, the parent has no bargaining power ($\theta = 1$), this mark-up is minimum, is equal to γL^2 and is only related to the preference of the young to have his parent closer. If the young had no preference for the distance ($\gamma = 0$), the mark up would even reduce to zero. The reason is the following. If $\theta = 1$, preferences of the parents, who only care about location, are not taken into account in the family decision process, and the preferences of the family correspond to those of children, who only want to minimize the price when $\gamma = 0$. Therefore, nursing homes cannot deviate from the marginal cost.

When interpreting Corollary 1, it should be reminded that the distribution of bargaining power can reflect various features. First, if the society strongly values the obedience of children to their parents (i.e. low θ), then the selection of a nursing home will only reflect the preferences of the parent, that is, the concern for the distance, and the price will not enter into the picture. In that case, nursing homes can charge a large mark up. If, on the contrary, the society strongly values the democracy within families, then the child will also have a word to say, and his interest for the price will reduce the capacity of nursing homes to extract a large mark up.

Alternatively, if one interprets the distribution of bargaining power as reflecting the degree of cognitive ability or awareness of the dependent parent, then Corollary 1 admits another interpretation. If the elderly's cognitive abilities are strongly limited, then the decision within the family will be made almost entirely by the child (i.e. θ close to 1). This limits the mark up of nursing homes. On the contrary, if the cognitive abilities of the dependent are still important, then the dependent parent will have more power in the nursing home decision, and as a consequence nursing homes will obtain higher margins.

4 Social optimum

Up to now, we focused only on an economy at the laissez-faire, that is, without state intervention. The laissez-faire equilibrium does not seem to be satisfactory from a social perspective, since this involves both (i) prices higher than marginal costs of production and (ii) large disutility for both children and their dependent parents, because of the extreme locations chosen by the nursing homes.

In this section, we first characterize the social optimum, and then discuss how it can be decentralized by means of policy instruments.

4.1 The centralized solution

Let us now turn to the social planning problem. For that purpose, we adopt a standard utilitarian social objective function, where the weights assigned to each individual's utility (i.e. children and dependent parents) are equal to 1/2.

The social planner chooses the locations of nursing homes, a and $L-b$, and prices p_A and p_B so as to maximize total welfare. With a uniform distribution of families on the line $[0, L]$ (and thus, with a density function $1/L$), its problem can be written as:

$$\begin{aligned} \max_{a,b,p_A,p_B} \quad W = & \int_{j=0}^{m} \left[\frac{1}{2}(w-p_A) - \frac{1}{2}(\gamma+\delta)(j-a)^2 \right] \frac{1}{L} dj \\ & + \int_{m}^{L} \left[\frac{1}{2}(w-p_B) - \frac{1}{2}(\gamma+\delta)(L-b-j)^2 \right] \frac{1}{L} dj \\ & \text{s.t.} \int_{j=0}^{m} p_A \frac{1}{L} dj + \int_{m}^{L} p_B \frac{1}{L} dj \geq \int_{0}^{L} c \frac{1}{L} dj \end{aligned} \tag{18}$$

where $m = m(a, b, p_A, p_B)$, the location of the median family, satisfies the condition:

$$\theta(w-p_A) - (\gamma\theta + (1-\theta)\delta)(m-a)^2 = \theta(w-p_B) - (\gamma\theta + (1-\theta)\delta)(L-b-m)^2. \tag{19}$$

Note that here, while the bargaining power does not appear explicitly in the planning problem, it is still implicitly present through the condition on the median agent, m.

In the Appendix, we show that whenever a (resp. b) increases, m goes further to the right (resp. to the left). When the price of nursing home A increases, the median goes further to the left meaning that more agents use nursing home B. The reverse reasoning applies when p_B increases.

We now derive the first order conditions of the above problem (see the Appendix) and making use of (19), these can be rewritten as follows:

$$\frac{\partial \mathcal{L}}{\partial p_A} = (\lambda - \frac{1}{2})m + \frac{dm}{dp_A}[p_B - p_A][\frac{1}{2}\frac{(1-2\theta)\delta}{\gamma\theta + (1-\theta)\delta} - \lambda] \leq 0 \quad (20)$$

$$\frac{\partial \mathcal{L}}{\partial p_B} = (\lambda - \frac{1}{2})m + \frac{dm}{dp_B}[p_B - p_A][\frac{1}{2}\frac{(1-2\theta)\delta}{\gamma\theta + (1-\theta)\delta} - \lambda] \leq 0 \quad (21)$$

$$\frac{\partial \mathcal{L}}{\partial a} = (\gamma + \delta)\int_{j=0}^{m}(j-a)dj + \frac{dm}{da}[p_B - p_A][\frac{1}{2}\frac{(1-2\theta)\delta}{\gamma\theta + (1-\theta)\delta} - \lambda] \leq 0 \quad (22)$$

$$\frac{\partial \mathcal{L}}{\partial b} = (\gamma + \delta)\int_{j=m}^{L}(L-b-j)dj + \frac{dm}{db}[p_B - p_A][\frac{1}{2}\frac{(1-2\theta)\delta}{\gamma\theta + (1-\theta)\delta} - \lambda] \leq 0 \quad (23)$$

where λ is the Lagrange multiplier associated with the resource constraint. The above first two conditions cannot be satisfied jointly unless $p_A^* = p_B^*$, since $dm/dp_A = -dm/dp_B$. Therefore, using the government budget constraint, it implies that prices should be equal to the marginal cost, c. Not surprisingly, prices at the social optimum are smaller than those obtained at the laissez faire and independent from bargaining powers.

Rearranging the last two conditions, one gets that:

$$a^* = \frac{m}{2}$$

$$b^* = \frac{L-m}{2}$$

This is clearly different from what we obtained at the laissez-faire equilibrium, where $a^{LF} = b^{LF} = 0$. This also implies that using condition (19) on the median family, it is such that it locates exactly in the middle of the line $[0, L]$, i.e. $m = m(a^*, b^*, c, c) = L/2$. Our results are summarized in Proposition 2.

Proposition 2 *At the utilitarian optimum, nursing homes A and B locate closer than at the laissez-faire on the line* $[0, L]$*:*

$$a^* = \frac{1}{4}L \text{ and } L - b^* = \frac{3}{4}L$$

and prices are equal to marginal costs:

$$p_A^* = p_B^* = c$$

The two nursing homes A and B equally share the demands: $D_A = D_B = m^* = L/2$.

Proof. See above. ■

The utilitarian optimum involves a quite different location of nursing homes in comparison to the laissez-faire. Contrary to the laissez-faire, where nursing homes A and B were located at the two extremes of the line (i.e. respectively at 0 and at L), the nursing homes are located, at the utilitarian optimum, in the middle of each half of the line, i.e. at $\frac{1}{4}L$ and $\frac{3}{4}L$. This more central location reduces strongly the average distance between nursing homes and visitors, and, hence, raises the utility of the child and of the dependent parent.

But this is not the unique source of improvement in social welfare. Another source lies in the reduction of prices in comparison with the laissez-faire. The extent to which the social optimum involves lower prices than the laissez-faire depends on the prevailing mark up at the laissez-faire, and, hence, on how the bargaining power is distributed within families.

Thus, the utilitarian social optimum involves welfare gains on two grounds: it makes nursing homes closer to families, and reduces the prices these have to pay for LTC services.

Note finally that at the social optimum, families end up having different utilities because of their distance to the nursing home. To avoid these utility inequalities, we could have assumed instead a social planner averse to inequality and thus, modelled it through a concave transformation of individual utilities in the social objective of problem (18). In the decentralised frameworks we are considering below, one would simply need to assume also lump sum transfers, proportional to the distance to the nursing home, to compensate agents for the disutility incurred by that distance.

4.2 Implementation

Let us study how the optimum described in Proposition 2 could be decentralized. For that purpose, we will proceed in two stages. We will first consider a first-best setting, where the government can impose their locations to nursing homes. Then, we will consider a second-best setting, where the government cannot impose locations.

4.2.1 Case A: locations can be forced

A first way of decentralizing this optimum consists in forcing locations of nursing homes A and B at a^* and at $L-b^*$. Additionally, nursing homes should be given a subsidy for each patient entering their nursing home. Let us denote s_i the subsidy received by nursing home i for each patient who is taken care of in this facility. Households decisions do not change. Only the problem faced by nursing homes is now different. Problem of Section 3.2 can thus be rewritten as

$$\max_{p_i} (p_i + s_i - c)D_i\left(p_i, p_{j\neq i}\right), \forall i = \{A, B\}$$

with p_i the prices faced by the patients going to nursing homes i. The first-order conditions for optimal prices are thus:

$$\begin{aligned} a + x_{m,A} + \frac{\partial x_{m,A}}{\partial p_A}(p_A + s_A - c) &= 0 \\ b + x_{m,B} + \frac{\partial x_{m,B}}{\partial p_B}(p_B + s_B - c) &= 0 \end{aligned}$$

Solving this system, one gets that

$$\begin{aligned} p_A^d &= -(\gamma\theta + (1-\theta)\,\delta)\frac{(a+b-L)(a-b+3L)}{3\theta} + (c - s_A) \qquad (24) \\ p_B^d &= -(\gamma\theta + (1-\theta)\,\delta)\frac{(-a+b+3L)(a+b-L)}{3\theta} + (c - s_B) \qquad (25) \end{aligned}$$

where d stands for decentralization. Equalizing these with the optimal prices levels, one gets that

$$\begin{aligned} s_A(a,b) &= (\gamma\theta + (1-\theta)\,\delta)\frac{(L-a-b)(a-b+3L)}{3\theta} \qquad (26) \\ s_B(a,b) &= (\gamma\theta + (1-\theta)\,\delta)\frac{(L-a-b)(-a+b+3L)}{3\theta} \qquad (27) \end{aligned}$$

which yields the following optimal values of the subsidies

$$s_A(a^*, b^*) = s_B^*(a^*, b^*) = (\gamma\theta + (1-\theta)\,\delta)\frac{L^2}{2\theta} \qquad (28)$$

The level of the subsidy is therefore the same for each nursing home. Note that this level is different from the mark up defined in Proposition 1, simply because locations are forced at different places than at the laissez faire. Since prices chosen by the facility depend on its location, these prices (in the absence of subsidization) would anyway be different from those set up at the laissez faire because location is different. The level of the subsidy is fixed so as to equalize prices to marginal costs.

For $\delta > 0$, the subsidy is therefore decreasing in the bargaining power, θ. The intuition is similar to that of the variation of the mark up with θ (see section 3.2.2). When θ is higher, i.e. the child has more bargaining power, the ability of the nursing home to deviate from the marginal cost and to charge high prices is more limited (since the child cares about the cost of the nursing home). Therefore, the optimal subsidy can be smaller.

From a social welfare point of view, it would then be optimal to foster the power of children. At the extreme, if $\theta \to 1$, the subsidy reduces to $\gamma L^2/2$, which corresponds to the compensation the government would have to give to facilities so that they locate at a^* and $L - b^*$. To the opposite, if $\theta \to 0$, s_i^* would be infinite.

4.2.2 Case B: locations cannot be forced

Let us now study how to decentralize the optimal allocation if the government cannot force the locations of nursing homes at a^* and $L - b^*$. To do so, we assume four instruments which lead the nursing homes to choose optimal prices and optimal locations. Like before, the problem of the family does not change, so that we will only consider the modified problems of nursing homes. We first assume that the government sets two subsidies, $s_A(a,b)$ and $s_B(a,b)$ so that prices are equalized to marginal costs. Obviously, the forms of the subsidies are identical to those defined in equations (26) and (27). Second, we assume that the government taxes nursing homes if they deviate from their optimal location. To do so, we assume a non-linear tax function imposed to each nursing home: $t_A = t\left(a - a^*\right)$ and $t_B = t\left(b - b^*\right)$ such that $t_i'(x) > 0 \forall x \in [0, L]$. In such a case, nursing home A chooses its location so as to maximize its profit, taking into account that it will be taxed if its location differs from the optimal one:

$$\begin{aligned}\max_a \pi^{A,d} &= (p_A^d + s_A(a,b)) - c)D_A(a,b) - t_A\left(a - a^*\right) \\ &= s_A(a,b)D_A(a,b) - t_A\left(a - a^*\right)\end{aligned}$$

Observe that the nursing home may have an interest in choosing a location a different from the optimal one so as to maximize $s_A(a,b)D_A$ with D_A defined by (10) and equal to $(L + a - b)/2$ at the decentralized allocation (since $p_A^d = p_B^d = c$).

First-order condition of this problem is:

$$\frac{\partial s_A(a,b)D_A(a,b)}{\partial a} - t_A'\left(a - a^*\right) = 0$$

so that the optimal marginal tax at $a = a^*$ should be equal to

$$t_A'(0) = \frac{\left(\gamma\theta + (1-\theta)\,\delta\right)}{6\theta}[3a^{*2} - 2a^*(b^* - 3L) - (b^* - L)^2]$$

Using the same reasoning for nursing home B, we obtain that the optimal level for the marginal tax at $b = b^*$ should be equal to:

$$t_B'(0) = \frac{\left(\gamma\theta + (1-\theta)\,\delta\right)}{6\theta}[3b^{*2} - 2b^*(a^* - 3L) - (a^* - L)^2]$$

Replacing for the value of a^* and b^*, we obtain

$$t_A'(0) = t_B'(0) = -\frac{\gamma\theta + \delta(1-\theta)}{6\theta}L^2 < 0. \tag{29}$$

Interestingly, nursing homes should be subsidized rather than taxed to locate at their optimal places a^* and $(L - b^*)$. In such a case, they equally share demands: $D_A = D_B = L/2$.

Our results are summarized in the following proposition.

Proposition 3 *The decentralization of the utilitarian social optimum can be attained through the following instruments:*

a) If location can be forced at a^* *and* $(L - b^*)$, *the government only needs to set subsidies given to each nursing home, equal to* (28).

b) If location cannot be forced, the government needs to set a non linear tax on the nursing homes if they deviate from the optimal location, in addition to the subsidies given to these nursing homes. The marginal tax at the optimal location is equal to (29).

Proof. See above. ■

Proposition 3 states that it is possible, thanks to adequate policy instruments, to decentralize the utilitarian social optimum, and to induce nursing homes to choose the optimal locations, and to charge the optimal prices. Note that, due to the fact that the mark up prevailing at the laissez-faire depends on the distribution of bargaining power within the family, it is also the case that the optimal values for subsidies depend on how the bargaining power is distributed, that is, on the level of θ. Thus, there is a strong link between, on the one hand, the optimal intervention of the State, and, on the other hand, how the family is structured in terms of decision power.

5 The size of the family

Up to now, we defined a family as a pair including one dependent parent and one child. This constitutes an obvious simplification. In this section, we propose to relax that assumption, in order to evaluate to what extent our results are robust to modifying the size of the family.

For that purpose, this section develops a variant model where families take the form of triplets including one dependent parent and two children.[14] For the sake of simplicity, we assume that the two children live at the same place on the line $[0, L]$. We assume that the two children, denoted 1 and 2, have different incomes, (w_1, w_2), as well as different preferences for the distance with respect to the nursing home, that are denoted by γ_1 and γ_2. We make no assumption on the ranking of w_1, w_2 and γ_1, γ_2.

The children share the cost of the nursing home as follows: child 1 pays a fraction β and child 2 pays $(1 - \beta)$ of the price of the nursing home $\{A, B\}$.

Both children have his word in the collective decision process regarding the choice of nursing home. Following the previous notations, we suppose that child 1 has a bargaining power θ_1, child 2 a bargaining power θ_2 and the dependent parent has a bargaining power $(1 - \theta_1 - \theta_2)$. Hence the utility of the family is:

$$\theta_1 \left[(w_1 - \beta p_i) - \gamma_1 x_i^2\right] + \theta_2 \left[(w_2 - (1 - \beta)p_i) - \gamma_2 x_i^2\right] + (1 - \theta_1 - \theta_2)\left(-\delta x_i^2\right) \quad (30)$$

Except the introduction of a second child, the structure of the model remains the same. Nursing homes choose their locations first, and, then, their prices. In

[14] As it is clear from the derivation of the problem below, we could have assumed any number of children without loss of generality.

a third stage, families choose the nursing home where the dependent parent is sent. Here again, the model is solved backwards.

5.1 Family decision

Let us first determine the demand for each nursing home. Taking back our model from the beginning, we identify the median family:

$$-(\theta_1\beta+\theta_2(1-\beta))p_A-\bar{D}x_{m,A}^2=-(\theta_1\beta+\theta_2(1-\beta))p_B-\bar{D}x_{m,B}^2$$

with $\bar{D}\equiv\theta_1\gamma_1+\theta_2\gamma_2+(1-\theta_1-\theta_2)\delta$, the average preference for the distance in the family. For the following, we also denote $\bar{\beta}\equiv\theta_1\beta+\theta_2(1-\beta)$.

Together with the constraint on distance (6), we obtain that:

$$x_{m,A} = \frac{\bar{\beta}(p_B-p_A)}{2(L-a-b)\bar{D}}+\frac{(L-a-b)}{2} \quad (31)$$

$$x_{m,B} = \frac{\bar{\beta}(p_A-p_B)}{2(L-a-b)\bar{D}}+\frac{(L-a-b)}{2} \quad (32)$$

which yields the following demands for nursing homes A and B respectively:

$$D_A(p_A,p_B) = \frac{\bar{\beta}(p_B-p_A)}{2(L-a-b)\bar{D}}+\frac{(L+a-b)}{2}$$

$$D_B(p_A,p_B) = \frac{\bar{\beta}(p_A-p_B)}{2(L-a-b)\bar{D}}+\frac{(L-a+b)}{2} \quad (33)$$

5.2 Nursing homes decisions

Profit maximization for nursing homes A and B yields:

$$p_A = -\bar{D}\frac{(a+b-L)(a-b+3L)}{3\bar{\beta}}+c \quad (34)$$

$$p_B = -\bar{D}\frac{(a+b-L)(-a+b+3L)}{3\bar{\beta}}+c \quad (35)$$

Proceeding as in Section 3.2.2, we find that, as before, nursing homes should locate at the extremes of the line, with $a^{LF}=0$ and $b^{LF}=L$, so that prices are equal in equilibrium to :

$$p_A^{LF}=p_B^{LF}=c+\frac{\theta_1\gamma_1+\theta_2\gamma_2+(1-\theta_1-\theta_2)\delta}{\theta_1\beta+\theta_2(1-\beta)}L^2 \quad (36)$$

The mark up rate (i.e. the last term above) therefore depends on the family average disutility from distance to the nursing home, and on the sharing rule of the LTC spending between the two children.

Let us first consider the impact of the average preference for the distance on the level of the mark up. It is easy to see that, as long as at least one of the family members has a strict preference for the distance and has some bargaining power, $\bar{D} > 0$ and the mark-up is positive.

Note also that, as in the baseline model, the size of the mark up depends strongly on how the bargaining power is distributed within the family. From that perspective, it has been argued, in Sloan et al. (1997), that a rise in the number of children is likely to raise the bargaining power of the dependent parent, since this reinforces the plausibility of a treat of leaving no bequest to the child. Within our model, this would translate in a larger bargaining power of the parent than in the baseline model (i.e. $1 - \theta_1 - \theta_2 > 1 - \theta$). This would then lead to a larger mark up in comparison to the baseline model.

Second, it should be stressed that the introduction of a second child has also another impact on the mark up, through the rule of LTC payment among children. Indeed, the denominator of the mark up expression is $\theta_1\beta + \theta_2(1 - \beta)$. This implies that, if the child that contributes the most to the LTC payment is not the one who has the largest bargaining power, this raises the mark up nursing homes can charge. This case may seem anecdotal, but is far from implausible, especially if children contribute according to their means.[15]

5.3 Social optimum and decentalisation

Assuming equal bargaining power between family members at the optimum, the optimal location and pricing are identical to what was found in Section 4:

$$\begin{aligned} a^* &= \frac{1}{4}L \text{ and } L - b^* = \frac{3}{4}L \\ p_A^* &= p_B^* = c \end{aligned}$$

Here again, the socially optimal location of nursing homes is far more central than the locations prevailing at the laissez-faire, whereas the optimal prices are also lower than at the laissez-faire.

The implementation is also identical, with :

$$s_A(a^*, b^*) = s_B^*(a^*, b^*) = \bar{D}\frac{L^2}{2\bar{\beta}} \tag{37}$$

when the government can force locations, and additional non linear taxation,

$$t'_A(0) = t'_B(0) = -\frac{\bar{D}}{6\bar{\beta}}L^2 < 0 \tag{38}$$

when it cannot force it. Proposition 4 summarizes our results.

[15] Take, for instance, the case of a child who succeeded in business, and can thus contribute a lot to the LTC spending, but has little time to bargain with the other family members. If the other child has been less successful (and has lower means to contribute), but has more time to bargain, it may be the case, at the end of the day, that the family is willing to pay a lot for the nursing home. As a consequence, this allows nursing homes to charge a larger mark up, in comparison to a situation where the main contributor would also have had more power in the family decision process.

Proposition 4 *Consider a variant of the baseline model, where each family is composed of one dependent parent and two children.*

At the laissez-faire, nursing homes A and B choose the locations:

$$a^{LF} = b^{LF} = 0$$

and prices are given by:

$$p_A^{LF} = p_B^{LF} = c + \frac{\theta_1\gamma_1 + \theta_2\gamma_2 + (1 - \theta_1 - \theta_2)\delta}{\theta_1\beta + \theta_2(1 - \beta)} L^2$$

At the utilitarian optimum, we have:

$$\begin{aligned} a^* &= \frac{1}{4}L \text{ and } L - b^* = \frac{3}{4}L \\ p_A^* &= p_B^* = c \end{aligned}$$

The decentralization uses the same instruments as in the baseline model.

Proof. See above. ■

Proposition 4 suggests that our results are robust to a rise in the size of the family, from one child to two children. It should be stressed, however, that this robustness is qualitative, but that, from a quantitative perspective, the size of the mark up of nursing homes may be significantly affected by changes in the family size. Various factors may indeed lead to levels of mark up rates that differ from what prevails in the case one parent / one child.

First, as already mentioned, the distribution of bargaining power may be quite different when the size of the family changes. If the rise in the number of children raises the bargaining power of the parent, this leads to larger mark up rates. But the opposite scenario may also arise: if the family takes its decisions on the basis of one person / one vote rule, implying $\theta_1 = \theta_2 = 1/3$, this may reduce the bargaining power of the parent, leading to lower mark up rates.

Another important feature that appears here is the role of the sharing rule for LTC costs. As we already mentioned, a dissonance between who has the power and who pays for LTC may favor a rise in mark up rates.

Furthermore, the introduction of another child may raise coordination failures. Whereas those failures are not modeled explicitly in our framework, it is possible to describe here their consequences on the mark up by changing the calibration of parameters. Suppose, for instance, that the introduction of a second child creates a coordination failure: each child relies on the other brother/sister for the visits of the dependent parent. In that case, from the perspective of each child, only the price of the nursing home matters, because the distance becomes irrelevant (since it is the other child who will make the visit). Hence, in that case, the intensity of the disutility for the distance would become quite low, i.e. $\gamma_1 = \gamma_2 \simeq 0$, which would strongly reduce the extent of the mark up rate charged by nursing homes. Thus the rise in the size of the family may not be neutral regarding the levels of mark up rates.

Finally, it should be stressed that, although it casts some light on the mechanisms at work, this extension remains, to some extent, limited by the assumption that all children live at the *same* place on the line $[0, L]$. Actually, once several children are present, one can suspect the occurrence of strategic location choices of children, in line with Konrad et al (2002) and Kureishi and Wakabayashi (2007). Note, however, that introducing different locations for children, and, *a fortiori*, strategic location choices for children would raise strong difficulties. Indeed, we assume here that nursing homes decide to locate on the basis of the demand (and thus of the geographical location of families). But once the locations of family members become endogenous as well, this becomes far from trivial to describe where family members and nursing homes will choose to locate.

6 Wealth accumulation and LTC price dynamics

Up to now, we considered a static economy with given resources. This constitutes a simplification, since, as the economy develops, this influences the resources available in the family, and, as a consequence, it may also affect the dynamics of nursing home prices. In order to study the relation between the dynamics of accumulation and the evolution of nursing home prices, this section considers a three-period OLG model.

6.1 The OLG economy

Each cohort is a continuum of agents of size L. Fertility is at the replacement level (one child per young agent). Period 1, whose duration is normalized to 1, is childhood, during which the child makes no decision. In period 2 (also of length 1), the agent is a young adult. He works in the production of goods, has one child, and saves a fraction $s \in]0, 1[$ of his resources, while he consumes a fraction $1 - s$ of his resources. In period 3, whose duration is $\lambda \in]0, 1[$, the individual is old and dependent, and is sent to a nursing home which is chosen by the dependent and his child through bargaining. When the parent dies, the share of the saved resources that are not spent in nursing home are transmitted to his child.

6.1.1 Production of LTC and of goods

The economy is now composed of two sectors: on the one hand, the production of LTC services by nursing homes (which takes place over a subperiod of size λ); on the other hand, the production of goods (which takes place over a period of unitary length).

For the sake of simplicity, the LTC sector is assumed to be the same as in the baseline model. It is a duopoly, with two nursing homes A and B. We suppose that the nursing home activity, which takes place only over a subperiod of size $\lambda < 1$, requires a quantity of good equal to c for each dependent person, as in the baseline model.[16] This quantity is purchased on the goods market, during a subperiod of size λ.

The production of goods is supposed to occur in a perfectly competitive sector. The production involves capital K_t and labor $L_t = L$ following a Cobb-Douglas production process:

$$Y_t = \phi K_t^{\alpha} L^{1-\alpha} \tag{39}$$

where Y_t denotes the output, and $\alpha \in [0,1]$. In intensive terms, we have:

$$y_t = \phi k_t^{\alpha} \tag{40}$$

where $y_t \equiv \frac{Y_t}{L}$ and $k_t \equiv \frac{K_t}{L}$ are the output and the capital stock per young adult.

We suppose a full depreciation of capital after one period of use.

Factors are paid at their marginal productivity:

$$w_t = \phi(1-\alpha)k_t^{\alpha} \tag{41}$$
$$R_t = \phi\alpha k_t^{\alpha-1} \tag{42}$$

where w_t is the wage rate, and R_t is one plus the interest rate.

6.1.2 Budget constraints

Each young individual has, as available resources, his wage w_t plus what he receives at the death of his parent (i.e. after a fraction of time λ). This amount is equal to the proceeds of the savings of the parent *minus* the LTC expenditure paid for his parent. Thus the available resources of the young are:

$$w_t + g_t - \lambda p_{it} \tag{43}$$

where g_t is the raw intergenerational transfer from the dead parent to the child, while λp_{it} is the cost of LTC. This is increasing in the price of the nursing home (which can be either A or B, as above), and also increasing in the duration of dependency λ.

Given that the young saves a fraction s of those resources, consumption at the young age is:

$$(1-s)\left(w_t + g_t - \lambda p_{it}\right) \tag{44}$$

Clearly, from the perspective of the child, the more costly the nursing home is, the lower the consumption at young age is.

The raw intergenerational transfer g_t coming from the parent is equal to the interest factor times the savings of the parent:

$$g_t = R_t s\left[w_{t-1} + g_{t-1} - \lambda p_{it-1}\right] \tag{45}$$

[16] Note that, if one wanted to make the LTC sector employ also labor and capital, the fact that the LTC activity stops when the dependent elderly die (i.e. after a period λ) would create a period of length $1-\lambda$ during which those factors would either be unemployed or reallocated towards the goods sector. By supposing that the LTC sector requires c units of good per dependent person, we abstract from those modelling difficulties.

This expression shows that the descending transfer from the parent to the child g_t depends on the transfer that the parent received, when he was young, from his own parent, i.e. g_{t-1}. Thus the model describes a dynamic of wealth accumulation. Obviously the expenditures in LTC tend to limit the scope of accumulation across generations.

6.1.3 Preferences

We suppose that individuals care, at the young age, about their consumption, and about how far they are from the nursing home of their parent (as in the previous sections). At the old age, individuals care, as above, about the distance between their nursing home and the location of their child. Moreover, old individuals now also care about the wealth they transmit to their child net of the price paid for the nursing home.

The lifetime utility of a young adult at time t is given by:

$$(1-s)\left(w_t+g_t-\lambda p_{it}\right)-\gamma\lambda x_{it}^2+\mu\left(R_{t+1}^e s\left(w_t+g_t-\lambda p_{it}\right)-\lambda p_{it+1}^e\right)-\delta\lambda x_{it+1}^{e2} \tag{46}$$

where the preference parameter $\mu \in [0,1]$ reflects the parent's interest in giving some wealth to his child net of the price paid for the nursing home. Note that the future interest factor is written in expectation terms, i.e. R_{t+1}^e. The same remark holds for the price of the future nursing home in which the agent will be at time $t+1$, i.e. p_{it+1}^e, and its distance from his own children, i.e. x_{it+1}^e. Note also that, given that the duration of the old age is here $\lambda < 1$, the disutility of distance is normalized by λ.

Using the same parameter θ to represent the bargaining power of the child, the utility of the family is now given by:

$$\begin{aligned}&\theta\left[\begin{array}{l}(1-s)\left(w_t+g_t-\lambda p_{it}\right)-\gamma\lambda x_{it}^2\\ +\mu\left(R_{t+1}^e s\left(w_t+g_t-\lambda p_{it}\right)-\lambda p_{lt+1}^e\right)-\delta\lambda x_{lt+1}^{e2}\end{array}\right]\\ &+(1-\theta)\left[\begin{array}{l}(1-s)\left(w_{t-1}+g_{t-1}-\lambda p_{jt-1}\right)-\gamma\lambda x_{jt-1}^2\\ +\mu\left(R_t s\left(w_{t-1}+g_{t-1}-\lambda p_{jt-1}\right)-\lambda p_{it}\right)-\delta\lambda x_{it}^{e2}\end{array}\right]\end{aligned} \tag{47}$$

Because of the OLG structure, the utility of the family depends on three nursing home choices. First, the wealth accumulated by the parent depends on the nursing home j where his own parent was sent. Second, the consumption of the child at the young age depends on the nursing home i where his parent is sent. Third, the transfer that the young agent will leave to his own child depends on the nursing home l where he will be sent once elderly and dependent.

6.2 Temporary equilibrium

At each period, the two nursing homes A and B choose their locations on $[0, L]$ and their prices. Moreover, each family chooses in which nursing home the dependent parent is sent. The timing is the same as above, and the problem is also solved by backward induction.

The only difference with respect to the baseline model is that all decisions are conditional on the available resources and production factor prices (wages and interest rates), and also conditional on expectations regarding future prices.

6.2.1 Family decision

As above, we solve the demand for each nursing home by first identifying the median family. For that particular family, the following equality prevails:

$$\begin{aligned} & \left[\begin{array}{l} \theta \left[\begin{array}{l} (1-s)\left(w_t+g_t-\lambda p_{At}\right)-\gamma\lambda x_{m,At}^2 \\ +\mu\left(R_{t+1}^e s\left(w_t+g_t-\lambda p_{At}\right)-\lambda p_{lt+1}^e\right)-\delta\lambda x_{lt+1}^{e2} \end{array} \right] \\ +(1-\theta)\left[\begin{array}{l} (1-s)\left(w_{t-1}+g_{t-1}-\lambda p_{jt-1}\right)-\gamma\lambda x_{jt-1}^2 \\ +\mu\left(R_t s\left(w_{t-1}+g_{t-1}-\lambda p_{jt-1}\right)-\lambda p_{At}\right)-\delta\lambda x_{m,At}^2 \end{array} \right] \end{array} \right] \\ = & \left[\begin{array}{l} \theta \left[\begin{array}{l} (1-s)\left(w_t+g_t-\lambda p_{Bt}\right)-\gamma\lambda x_{m,Bt}^2 \\ +\mu\left(R_{t+1}^e s\left(w_t+g_t-\lambda p_{Bt}\right)-\lambda p_{lt+1}^e\right)-\delta\lambda x_{lt+1}^{e2} \end{array} \right] \\ +(1-\theta)\left[\begin{array}{l} (1-s)\left(w_{t-1}+g_{t-1}-\lambda p_{jt-1}\right)-\gamma\lambda x_{jt-1}^2 \\ +\mu\left(R_t s\left(w_{t-1}+g_{t-1}-\lambda p_{jt-1}\right)-\lambda p_{Bt}\right)-\delta\lambda x_{m,Bt}^2 \end{array} \right] \end{array} \right] \end{aligned}$$

which simplifies to:

$$\begin{aligned} & \theta\left[(1-s)\left(-p_{At}\right)-\gamma x_{m,At}^2+\mu\left(R_{t+1}^e s\left(-p_{At}\right)\right)\right]+(1-\theta)\left[\mu\left(-p_{At}\right)-\delta x_{m,At}^2\right] \\ = \; & \theta\left[(1-s)\left(-p_{Bt}\right)-\gamma x_{m,Bt}^2+\mu\left(R_{t+1}^e s\left(-p_{Bt}\right)\right)\right]+(1-\theta)\left[\mu\left(-p_{Bt}\right)-\delta x_{m,Bt}^2\right]. \end{aligned}$$

Hence

$$\left(x_{m,Bt}^2-x_{m,At}^2\right)(\gamma\theta+\delta(1-\theta))=\left(p_{At}-p_{Bt}\right)\left[\theta(1-s)+\theta\mu s R_{t+1}^e+(1-\theta)\mu\right] \tag{48}$$

Using $x_{m,At}+x_{m,Bt}=L-a_t-b_t$, we have:[17]

$$x_{m,At} = \frac{\left(p_{Bt}-p_{At}\right)\left[\theta(1-s)+\theta\mu s R_{t+1}^e+(1-\theta)\mu\right]}{2\left(L-a_t-b_t\right)(\gamma\theta+\delta(1-\theta))}+\frac{\left(L-a_t-b_t\right)}{2} \tag{49}$$

$$x_{m,Bt} = \frac{\left(p_{At}-p_{Bt}\right)\left[\theta(1-s)+\theta\mu s R_{t+1}^e+(1-\theta)\mu\right]}{2\left(L-a_t-b_t\right)(\gamma\theta+\delta(1-\theta))}+\frac{\left(L-a_t-b_t\right)}{2} \tag{50}$$

Obviously, if $\mu = s = 0$, those expressions are the same as in the baseline model without wealth transmission. In that case, the unique opportunity cost of LTC expenditures is to reduce the consumption of the young. However, in the more general case where $\mu, s > 0$, the opportunity cost of LTC is threefold, and involves not only a reduction of the consumption of the young to an extent $1-s$, but also two other effects related to wealth transmission. First, LTC spending reduces, proportionally to s, the amount of wealth that can be transmitted from the child of the dependent to his own child. The extent to which LTC spending reduce the size of intergenerational transfers depends on how large the interest

[17] Note that the location choice variables a_t and b_t are here indexed with time, since it cannot be excluded, at this stage, that the optimal location of nursing homes varies over time.

factor R^e_{t+1} is. Second, the cost of LTC also reduces the wealth transfer that the old gave to his child, which matters for the utility of the elderly.

Assuming myopic anticipations (i.e. $R^e_{t+1} = R_t$), we can write the demands for the two nursing homes as:

$$D_{At}(p_{At}, p_{Bt}) = a_t + \frac{(p_{Bt} - p_{At})\left[\theta(1-s) + \theta\mu s R_t + (1-\theta)\mu\right]}{2(L - a_t - b_t)(\gamma\theta + \delta(1-\theta))} + \frac{(L - a_t - b_t)}{2} \tag{51}$$

$$D_{Bt}(p_{At}, p_{Bt}) = b_t + \frac{(p_{At} - p_{Bt})\left[\theta(1-s) + \theta\mu s R_t + (1-\theta)\mu\right]}{2(L - a_t - b_t)(\gamma\theta + \delta(1-\theta))} + \frac{(L - a_t - b_t)}{2} \tag{52}$$

A higher interest factor makes the demand more reactive to prices, since this raises the opportunity cost of LTC spending. As a consequence of this, the level of the interest rate will also limit the capacity of nursing homes to extract large rents.

6.2.2 Nursing homes decisions

Throughout this section, we consider the choices of prices and locations by nursing homes. In order to avoid a too large departure with respect to our baseline model, we assume that nursing homes have a limited time horizon, in the sense that their objective at a given period is to maximize its profits at that same period.

Setting prices Let us first consider the choice of prices conditionally on the nursing homes' location.[18]

The problem faced by nursing home A at time t is:

$$\max_{p_{At}} (p_{At} - c)\, D_{At}(p_{At}, p_{Bt})$$

with $D_{At}(p_{At}, p_{Bt})$ given by (51). The FOC yields:

$$p_{At} = \frac{c}{2} + \frac{p_{Bt}}{2} + \frac{(L - a_t - b_t)(\gamma\theta + \delta(1-\theta))}{\left[\theta(1-s) + \theta\mu s R_t + (1-\theta)\mu\right]} \frac{(L + a_t - b_t)}{2}$$

The problem faced by nursing home B at time t is:

$$\max_{p_{Bt}} (p_{Bt} - c)\, D_{Bt}(p_{At}, p_{Bt})$$

The FOC yields:

$$p_{Bt} = \frac{c}{2} + \frac{p_{At}}{2} + \frac{(L - a_t - b_t)(\gamma\theta + \delta(1-\theta))}{\left[\theta(1-s) + \theta\mu s R_t + (1-\theta)\mu\right]} \frac{(L - a_t + b_t)}{2}$$

[18] Here LTC prices are prices expressed in terms of goods.

Hence, solving for optimal prices, we have:

$$p_{At} = c + \frac{(L - a_t - b_t)(\gamma\theta + \delta(1-\theta))}{3[\theta(1-s) + \theta\mu R_t s + (1-\theta)\mu]}[3L + a_t - b_t] \quad (53)$$

$$p_{Bt} = c + \frac{(L - a_t - b_t)(\gamma\theta + \delta(1-\theta))}{3[\theta(1-s) + \theta\mu R_t s + (1-\theta)\mu]}[3L - a_t + b_t] \quad (54)$$

where the last terms are on the RHS of the above equations are the mark up rates imposed by nursing homes. As mentioned previously, the higher the interest factor is, the lower the mark up charged by nursing homes is. Indeed, a higher interest factor raises the opportunity cost of spending on LTC for the child, since this prevents, to a larger extent, wealth transmission to the next generation. The intuition is that a higher interest factor makes the demand more reactive to prices, since this raises the opportunity cost of LTC spending. Hence, this limits the extent of the mark up for nursing homes.

Substituting for those prices in the location of the nursing homes with respect to the median family, we obtain:

$$x_{m,At} = \frac{3L - 5a_t - b_t}{6} \quad (55)$$

$$x_{m,Bt} = \frac{3L - a_t - 5b_t}{6} \quad (56)$$

These are the same expressions as in the baseline model, except that locations are indexed by time.

Choosing locations Let us now consider the choices of location for the two nursing homes A and B at time t, conditionally on the optimal prices derived above.

The problem of nursing home A is:

$$\max_{a_t} \frac{(L - a_t - b_t)(\gamma\theta + \delta(1-\theta))}{18[\theta(1-s) + \theta\mu R_t s + (1-\theta)\mu]}[3L + a_t - b_t]^2$$

The FOC is:

$$\frac{(\gamma\theta + \delta(1-\theta))[3L + a_t - b_t]}{18[\theta(1-s) + \mu\theta R_t s + (1-\theta)\mu]}[-L - 3a_t - b_t] < 0$$

Thus it is optimal for nursing home A to choose $a_t = 0$, that is, to locate at the extreme left of the segment $[0, L]$.

The problem of nursing home B is:

$$\max_{b_t} \frac{(L - a_t - b_t)(\gamma\theta + \delta(1-\theta))}{18[\theta(1-s) + \theta\mu R_t s + (1-\theta)\mu]}[3L - a_t + b_t]^2$$

The FOC is:

$$\frac{(\gamma\theta + \delta(1-\theta))[3L - a_t + b_t]}{18[\theta(1-s) + \theta\mu R_t s + (1-\theta)\mu]}[-L - a_t - 3b_t] < 0$$

Thus the nursing home B chooses $b_t = 0$, that is, to locate at L, i.e. at the extreme right of the segment $[0, L]$.

The following proposition summarizes our results.

Proposition 5 *At the temporary equilibrium under myopic anticipations, the two nursing homes locate at the far extreme of the line* $[0, L]$*, independently of the distribution of bargaining power within the family.*

Prices in the two nursing homes are equal to:

$$p_{At} = p_{Bt} = c + \frac{(\gamma\theta + \delta(1-\theta))}{[\theta(1-s) + \theta\mu R_t s + (1-\theta)\mu]} L^2$$

The demand for each nursing home is $D_{At} = D_{Bt} = L/2$.

Proof. See above. ■

At the temporary equilibrium, the principle of maximum differentiation holds. Each nursing home remains located at the two extremes of the segment $[0, L]$, as in the baseline model. However, in comparison with the baseline model, the formulae for prices are here different, to an extent that depends on how large the propensity to save s is, on how much parents care about transmitting wealth (i.e. the level of μ), and on the interest factor R_t. The reason why those factors influence the above prices lies in the fact that wealth accumulation matters for individuals. The price of nursing homes determines the size of LTC expenditure, which in turn, limit capital accumulation.

For a given distribution of bargaining power (i.e. a given θ), the price of nursing homes is decreasing with μ, that is, with the intensity of the individual preference for transmitting wealth to his child. Thus, individual's willingness to transmit wealth limits the extent to which nursing homes can realize a high mark-up. This limitation in the mark up is even larger when the interest factor R_t is larger.

Another important difference with respect to the baseline model concerns the impact of the distribution of bargaining power on the mark up of nursing homes, as stated in the following corollary.

Corollary 2 *At the temporary equilibrium with myopic anticipations, the mark up of nursing homes varies non-monotonically with the bargaining power of the child in the family:*

$$\frac{dMarkup}{d\theta} = \frac{-\delta(1-s) - s\delta\mu R_t + \gamma\mu}{[\theta(1-s) + \theta\mu R_t s + (1-\theta)\mu]^2} L^2$$

We have

$$\frac{dMarkup}{d\theta} < 0 \iff -\delta(1-s) - s\delta\mu R_t + \gamma\mu < 0$$

Proof. This is obtained by taking the derivative of $\frac{(\gamma\theta+\delta(1-\theta))}{[\theta(1-s)+\theta\mu R_t s+(1-\theta)\mu]} L^2$ with respect to θ. ■

That result is quite different from what prevailed in the baseline model, where a larger bargaining power for children in the family had the unambiguous effect to reduce the margins of nursing homes. Indeed, when $s = \mu = 0$, we have $\frac{dMarkup}{d\theta} = \frac{-\delta}{[\theta]^2} L^2 < 0$. This is not necessarily the case here. Indeed, there are now three effects at play. On the one hand, increasing prices reduces consumption of the young, up to their propensity to consume $(1 - s)$. It also reduces the amount of resources, he can transmit to his child. For these two reasons, this limits the capacity of nursing homes to charge high prices so that when the bargaining power of the child increases, the mark up charged by nursing homes have to decrease. This is reflected by the two first terms, $-\delta(1 - s) - s\delta\mu R_t$ in the above inequality. On the other hand, the parent gets lower net utility from transmitting wealth if prices increase. But as θ increases, his bargaining power decreases and nursing homes can charge higher prices. This last effect is reflected through $\gamma\mu$ in the above equality and is positive. Depending on the magnitude of these three effects, it may then be the case that the mark up increases with θ (for instance if γ is high).

Corollary 2 suggests that the results obtained in the baseline model were not fully robust to how we specify the interests of the dependent parents. True, when these are only concerned about the distance between the nursing home and the child, a higher bargaining power for parents will necessarily raise the mark up for nursing homes. However, once parents also care about the amount they transmit to their children net of the nursing home price, this result does not necessarily hold anymore.

Finally, it should be stressed that the relation between the distribution of bargaining power in the family and the mark up of nursing homes depends on the level of the interest factor R_t. When this is high, it is likely that the mark up remains decreasing with the bargaining power of the children. However, when it is low, it may be the case that the mark up is increasing with the bargaining power of the children (this is actually the case when $\gamma\mu > \delta(1 - s) + s\delta\mu R_t$). Hence, in order to study the dynamics of nursing home prices, it is necessary to study its links with the dynamics of wealth accumulation as we do in the following section.

6.3 Intertemporal equilibrium

The capital accumulation follows the law:

$$k_{t+1} = s\left(w_t + g_t - \lambda p_t\right) \tag{57}$$

where $g_t = R_t s_{t-1} = R_t s\left[w_{t-1} + g_{t-1} - \lambda p_{t-1}\right]$. The capital accumulation equation shows that high nursing homes prices prevent capital accumulation given the fixed propensity to save.[19] Similarly the equation for transfer g_t shows that high nursing home prices prevent wealth accumulation.

[19] Note that this result follows from the assumption that all profits of the nursing homes are spent on the good market. Alternatively, if we had supposed that all the profits were saved, then high nursing homes prices would have favoured capital accumulation.

Hence the economy can be described by the following system:

$$\begin{aligned} k_{t+1} &= s\left(w_t + g_t - \lambda p_t\right) \\ g_{t+1} &= R_{t+1}s\left[w_t + g_t - \lambda p_t\right] \\ p_{t+1} &= c + \frac{(\gamma\theta + \delta(1-\theta))}{[\theta(1-s) + \theta\mu s R_{t+1} + (1-\theta)\mu]}L^2 \end{aligned}$$

Note that, from the first two relations, it appears that $g_{t+1} = R_{t+1}k_{t+1} = \phi\alpha\left(k_{t+1}\right)^{\alpha-1}k_{t+1} = \phi\alpha k_{t+1}^{\alpha}$. Hence, substituting for $g_t = \phi\alpha k_t^{\alpha}$, $R_t = \phi\alpha k_t^{\alpha-1}$ and $w_t = \phi(1-\alpha)k_t^{\alpha}$, the system can be reduced to a two-dimensional system:

$$\begin{aligned} k_{t+1} &= s\left(\phi(1-\alpha)k_t^{\alpha} + \phi\alpha k_t^{\alpha} - \lambda p_t\right) = s\left(\phi k_t^{\alpha} - \lambda p_t\right) \\ p_{t+1} &= c + \frac{(\gamma\theta + \delta(1-\theta))}{\left[\theta(1-s) + \theta\mu s\phi\alpha\left[s\left(\phi k_t^{\alpha} - \lambda p_t\right)\right]^{\alpha-1} + (1-\theta)\mu\right]}L^2 \end{aligned}$$

A stationary equilibrium is defined as a situation where the economy perfectly reproduces itself over time. More formally, a stationary equilibrium is a pair (k_t, p_t) such that $k_{t+1} = k_t = k$ and $p_{t+1} = p_t = p$.

The following proposition examines the issues of existence and uniqueness of a stationary equilibrium in our economy.

Proposition 6 *Denote* $\Omega \equiv \theta(1-s) + (1-\theta)\mu$ *and* $\Lambda \equiv (\gamma\theta + \delta(1-\theta))\,L^2$.

Suppose that $(p_t - c)\left[\Omega + \theta\mu s^{\alpha}\phi\alpha\left(-\lambda\right)^{\alpha-1}\left(p_t\right)^{\alpha-1}\right] = \Lambda$ *has a unique solution* $p_t > 0$ *denoted by* $\bar{p}$.

Suppose that $(p_t - c)\left[\Omega + \theta\mu s^{\alpha}\phi\alpha\left(\phi\left(s\alpha\phi\right)^{\frac{\alpha}{1-\alpha}} - \lambda p_t\right)^{\alpha-1}\right] = \Lambda$ *has a unique solution* $p_t > 0$ *denoted by* $\tilde{p}$.

Suppose that $(p_t - c)\left[\Omega + \theta\mu s^{\alpha}\phi\alpha\left(\phi\left(s\phi\right)^{\frac{\alpha}{1-\alpha}} - \lambda p_t\right)^{\alpha-1}\right] = \Lambda$ *has a unique solution* $p_t > 0$ *denoted by* $\hat{p}$.

then the condition

$$\frac{s\phi\left(s\alpha\phi\right)^{\frac{\alpha}{1-\alpha}} - \left(s\alpha\phi\right)^{\frac{1}{1-\alpha}}}{\lambda s} > \tilde{p}$$

is sufficient to guarantee that there exists at least two stationary equilibria (k_1, p_1) *and* (k_2, p_2) *with:*

$$\begin{aligned} 0 &< k_1 < (s\alpha\phi)^{\frac{1}{1-\alpha}} < k_2 < (s\phi)^{\frac{1}{1-\alpha}} \\ p_1 &< p_2 \end{aligned}$$

Proof. See the Appendix. ■

Proposition 6 suggests that, in our simple OLG economy, there exists, under plausible conditions, at least two stationary equilibria. We know for sure that there exists a positive correlation between, on the one hand, the sustainable level of capital, and, on the other hand, the price of nursing home associated to those stationary equilibria. In other words, Proposition 6 states that richer stationary economies are also characterized by higher nursing home prices. Inversely, poorer economies are characterized by lower LTC prices. The intuition behind that result lies in the fact that the mark up rate can be higher in richer economies, where the interest factor is lower.

Whereas Proposition 6 informs us about the existence and non-uniqueness of a stationary equilibrium in our economy, it tells us nothing about the stability of those equilibria. Stability is examined in Proposition 7.

Proposition 7 *Considering the stability of stationary equilibria* (k_1, p_1) *and* (k_2, p_2)*, two cases can arise:*

- *If, for a stationary equilibrium level* k_r*, we have*

$$\left| s\phi\alpha k^{\alpha-1} + \frac{\Lambda\lambda\mu s^{\alpha}\theta\phi\alpha\left(\alpha-1\right)k^{\alpha-2}}{\left[\mu s^{\alpha}\theta\phi\alpha k^{\alpha-1}+\Omega\right]^{2}} \right| < 1$$

  then the stationary equilibrium (k_r, p_r) *is locally stable.*

- *If, for a stationary equilibrium level* k_r*, we have*

$$\left| s\phi\alpha k^{\alpha-1} + \frac{\Lambda\lambda\mu s^{\alpha}\theta\phi\alpha\left(\alpha-1\right)k^{\alpha-2}}{\left[\mu s^{\alpha}\theta\phi\alpha k^{\alpha-1}+\Omega\right]^{2}} \right| > 1$$

  then the stationary equilibrium (k_r, p_r) *is a saddle point (and is thus unstable).*

- *Assuming* $\alpha = \frac{1}{2}$ *and* $c = 0$*, the condition for local stability for* (k_1, p_1) *is:*

$$\left| \frac{\Omega s\phi}{-\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right) + \Psi} - \left[\frac{1}{4} \frac{\Lambda\mu s^{2}\theta\phi\left[\frac{2\Omega}{\left[-\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)+\Psi\right]}\right]^{3}}{\left[\frac{\Omega\mu s^{1/2}\theta\phi}{\left[-\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)+\Psi\right]}+\Omega\right]^{2}} \right] \right| < 1$$

  whereas the condition for stability for (k_2, p_2) *is:*

$$\left| \frac{\Omega s\phi}{-\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right) - \Psi} - \left[\frac{1}{4} \frac{\Lambda\mu s^{2}\theta\phi\left[\frac{2\Omega}{\left[-\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)-\Psi\right]}\right]^{3}}{\left[\frac{\Omega\mu s^{1/2}\theta\phi}{\left[-\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)-\Psi\right]}+\Omega\right]^{2}} \right] \right| < 1$$

  where $\Psi \equiv \sqrt[2]{\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right)^{2} - 4\Omega\left(\lambda s\Lambda - \frac{(s\phi)^{2}\mu\theta}{2}\right)}$.

Proof. See the Appendix. ■

As it can be seen from the first part of Proposition 7, it is hard, without imposing further constraints on some parameters, to assess the plausibility of the stability conditions, since these depend on the prevailing level of k at the stationary equilibrium, for which there is no closed form solution. This is the reason why the second part of Proposition 7 imposes further parametric restrictions, and derives explicit stability conditions for the two stationary equilibrium, conditions that depend only on the structural parameters of the economy.

Those conditions are hard to interpret, but a clear message is that the possibility of a saddle point equilibrium can hardly be excluded *a priori*. As a consequence, it is possible, in this model, that there exists some unstable stationary equilibria, implying, for given initial conditions, that both wealth accumulation and the price of nursing homes may not converge, in the long run, towards some stable values, but rather keep on diverging further from the stationary equilibria. Thus it cannot be excluded that initial conditions matter for the evolution of both wealth accumulation and LTC prices.

Note, here again, that whether the stationary equilibria are stable or not is not independent from how the bargaining power is distributed within the family. To see this, take the condition for the stability of the high stationary equilibrium under $\alpha = \frac{1}{2}$ and $c = 0$. Clearly, when parents have the entire decision power, we have $\theta = 0$ and the condition for the high stationary equilibrium (k_2, p_2) simplifies to:

$$\left| \frac{s\phi\mu}{s\phi\mu - \sqrt[2]{(s\phi\mu)^2 - \mu\lambda s 4\delta L^2}} \right| > 1$$

Thus, in that case the stability condition is clearly violated, and the high stationary equilibrium is a saddle point. Hence, under that parametrization, an economy where the dependent parent has the entire decision power is unlikely to converge, except for particular initial conditions (making the economy on the saddle path), towards the high stationary equilibrium.

On the contrary, the stability condition for the low stationary equilibrium, (k_1, p_1) which simplifies here to:

$$\left| \frac{s\phi\mu}{s\phi\mu + \sqrt[2]{(s\phi\mu)^2 - \mu\lambda s 4\delta L^2}} \right| < 1$$

is satisfied, implying that the low stationary equilibrium, which involves a lower level of sustainable capital and lower nursing home prices, is locally stable. Thus, as the above example shows, the distribution of bargaining power within the family matters for the stability of the high and low stationary equilibria, and, hence, for the whole long-run dynamics of wealth and LTC prices.

7 Conclusions

This paper proposed to study interactions between, on the one hand, the distribution of bargaining power within the family, and, on the other hand, the prices and location of nursing homes. For that purpose, we developed several variants of the Hotelling model of spacial competition between nursing homes, where the demand for nursing homes is the outcome of a family bargaining process.

Our main findings are that, first of all, the mark up rate of nursing homes is increasing with the bargaining power of the dependent parent. This result was shown to be robust to the extension of the size of the family. The underlying intuition is that, if only the child or the children care about the price of nursing home (and not the dependent person), a higher bargaining power for the dependent parent leads to both a larger weight given to the distance dimension and a lower weight to the price dimension. It therefore allows nursing homes to charge larger mark up rates. Note, however, that this result does not necessarily hold in an OLG setting where parents care about the transmission of wealth to their child (net of the nursing home price). In that case, parents may also want to avoid paying too large prices for nursing homes, since high LTC expenditures prevent wealth transmission towards the descendants.

Another important finding concerns the comparison of the laissez-faire equilibrium with the utilitarian social optimum. Clearly, whereas the laissez-faire involves nursing homes located at the two extreme of the line (following the principle of maximal differentiation), the social optimum involves nursing homes located in a more central manner, in the middle of the two halves of the line. This implies that the disutility of the distance is, for the dependent parents and the children, significantly reduced at the utilitarian optimum, which also involves lower prices (the mark up being reduced to zero at the optimum). We examined how that social optimum could be decentralized, even in a second-best world where governments cannot force the location of nursing homes.

Our extension to an OLG economy allowed us to emphasize that the dynamics of wealth and of LTC prices are related through various channels. First, high LTC prices, by reducing, under a fixed propensity to save, the size of descending wealth transfers, prevent capital accumulation. But the relationship goes also in the other way: a higher capital stock reduces the interest rate, which decreases the opportunity cost of LTC spending. As a consequence, a higher capital stock raises the price of nursing homes through higher mark up rates.

Moreover, our analysis of the joint dynamics of wealth accumulation and LTC prices allowed us also to emphasize the crucial role played, here again, by the distribution of bargaining power within the family. A multiplicity of equilibria can easily arise, and stability is not guaranteed. Some saddle points can arise, making convergence towards a stationary equilibrium quite unlikely. Stability conditions are clearly influenced by how the bargaining power is distributed within families, and this suggests that the impact of bargaining power distribution is not only temporary, but can also drive the whole long run dynamics of capital accumulation and nursing home prices.

8 References

Bester, H. (1989). Noncooperative Bargaining and Spatial Competition, *Econometrica*, 57, 97-113. Brekke, K., Siciliani, L., Straume, O.R. (2014). Can competition reduce quality? CESifo Discussion Paper 4629.

Brown, J., Coe, N., Finkelstein, A. (2007). Medicaid Crowd-Out of Private Long-Term Care Insurance Demand: Evidence from the Health and Retirement Survey, NBER Chapters, in: *Tax Policy and the Economy*, Volume 21, pages 1-34 National Bureau of Economic Research, Inc.

Brown, J., Finkelstein, A. (2009). The Private Market for Long Term Care Insurance in the United States: A Review of the Evidence, *Journal of Risk and Insurance*, 76, 5-29.

Brown, J., Finkelstein, A. (2011). Insuring long term care in the U.S., *Journal of Economic Perspectives*, 25, 119-142.

Cremer, H., Pestieau, P, Ponthiere, G. (2012). The Economics of Long Term Care: a Survey, *Nordic Economic Policy Review*, 2, 107-148.

de la Croix, D., Vander Donkt, M. (2010). Would empowering women initiate the demographic transition in least developed countries? *Journal of Human Capital*, 4 (2), 85-129.

EU (2015). *The 2015 Ageing Report*, European Economic 3.

Hiedemann, B., Stern, S. (1999). Strategic play among family members when making long term care decisions. *Journal of Economic Behavior and Organization*, 40, 29-57.

Hoerger, T.J., Picone, G., Sloan, F. (1996). Public subsidies, private provision of care and living arrangements. *Review of Economics and Statistics*, 78, 428-440.

Hotelling, H. (1929). Stability in competition. *Economic Journal*, 39, 41-57.

Jousten, A., Lipszyc, B., Marchand, M., Pestieau, P. (2005). Long-term care insurance and optimal taxation for altruistic children. *FinanzArchiv*, 61, 1-18.

Konrad, K. A., Kunemund, H., Lommerud, K. E. , Robledo, J.R. (2002). Geography of the Family. *American Economic Review*, 92(4): 981–998.

Konrad, K. A., Lommerud, K. E. (2000). The bargaining family revisited. *Canadian Journal of Economics*, 33 (2), 471-487.

Kureishi, W., Wakabayashi, M. (2007). Why do First-born Children Live with Parents? — Geography of the Family in Japan —. Osaka University, mimeo.

Leker, L., Ponthiere, G. (2015). Education, life expectancy and family bargaining: the Ben-Porath effect revisited. *Education Economics*, 23 (4), 481-513.

Norton, E. (2000): Long term care, in A. Cuyler & J. Newhouse (eds.): *Handbook of Health Economics*, Volume 1, chapter 17.

Pestieau P., Sato, M. (2008). Long term care: the State, the Market and the Family, *Economica*, 75, 435-454.

Pezzin, L., Pollak, R., Schone, B. (2007). Efficiency in family bargaining: living arrangements and caregiving decisions of adult children and disabled elderly parents. *CESifo Economic Studies*, 53, 69-96.

Pezzin, L., Pollak, R., Schone, B. (2009). Long term care of the disabled elderly: do children increase caregiving by spouses? *Review of Economics of the Household*, 7, 323-339.

Schmitz H., Stroka, M. (2014). Do the Elderly Choose nursing Homes by Quality, Price or Location, RHUR Economic Papers 495.

Sloan, F., Picone, G., Hoerger, T. (1997). The supply of children 's time to disabled elderly parents. *Economic Inquiry*, 35, 295-308.

Stern, S., Engers, M. (2002). LTC and family bargaining, *International Economic Review*, 43(1), 73-114.

9 Appendix

9.1 The social optimum

Fully differentiating condition (19) with respect to (a, b, p_A, p_B), one can show that:

$$\begin{aligned}
\frac{dm}{da} &= \frac{m-a}{L-b-a} > 0 \\
\frac{dm}{db} &= \frac{m-(L-b)}{L-b-a} < 0 \\
\frac{dm}{dp_A} &= -\frac{\theta}{2(\gamma\theta + (1-\theta)\delta)\,(L-b-a)} < 0 \\
\frac{dm}{dp_B} &= \frac{\theta}{2(\gamma\theta + (1-\theta)\delta)\,(L-b-a)} > 0
\end{aligned}$$

Problem (18) yields the following rearranged first order conditions.

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial p_A} &= \int_{j=0}^{m} (\lambda - \frac{1}{2})dj + \frac{1}{2}\frac{dm}{dp_A} \\
&\quad \times [p_B - p_A + (\gamma+\delta)((L-b-m)^2 - (m-a)^2)] \\
&\quad + \lambda(p_A - p_B)\frac{dm}{dp_A} \leq 0 \qquad (58) \\
\frac{\partial \mathcal{L}}{\partial p_B} &= \int_{j=0}^{m} (\lambda - \frac{1}{2})dj + \frac{1}{2}\frac{dm}{dp_B} \\
&\quad \times [p_B - p_A + (\gamma+\delta)((L-b-m)^2 - (m-a)^2)] \\
&\quad + \lambda(p_A - p_B)\frac{dm}{dp_B} = 0 \qquad (59) \\
\frac{\partial \mathcal{L}}{\partial a} &= (\gamma+\delta)\int_{j=0}^{m} (j-a)dj + \frac{1}{2}\frac{dm}{da} \\
&\quad \times [(p_B - p_A) + (\gamma+\delta)((L-b-m)^2 - (m-a)^2)] \\
&\quad + \lambda(p_A - p_B)\frac{dm}{da} \leq 0 \qquad (60) \\
\frac{\partial \mathcal{L}}{\partial b} &= (\gamma+\delta)\int_{j=m}^{L} (L-b-j)dj + \frac{1}{2}\frac{dm}{db} \\
&\quad \times [(p_B - p_A) + (\gamma+\delta)((L-b-m)^2 - (m-a)^2)] \\
&\quad + \lambda(p_A - p_B)\frac{dm}{db} \leq 0 \qquad (61)
\end{aligned}$$

9.2 Proof of Proposition 6

The dynamics of the economy are described by:

$$\begin{aligned} k_{t+1} &= s\left(\phi(1-\alpha)k_t^{\alpha} + \phi\alpha k_t^{\alpha} - \lambda p_t\right) = s\left(\phi k_t^{\alpha} - \lambda p_t\right) \\ p_{t+1} &= c + \frac{(\gamma\theta + \delta(1-\theta))}{\left[\theta(1-s) + \mu s\theta\phi\alpha\left[s\left(\phi k_t^{\alpha} - \lambda p_t\right)\right]^{\alpha-1} + (1-\theta)\mu\right]} L^2 \end{aligned}$$

Let us define the kk locus, along which capital per worker is constant. This is defined by the relation:

$$k_t = s\left(\phi k_t^{\alpha} - \lambda p_t\right)$$

Isolating p_t, we obtain:

$$p_t = \frac{s\phi k_t^{\alpha} - k_t}{\lambda s} \tag{62}$$

Let us focus on the (k_t, p_t) space (see Figure 1 at the end of this proof). The kk locus intersects the x axis at $k_t = 0$ and when $s\phi k_t^{\alpha} - k_t = 0$, that is, when $k_t = (s\phi)^{\frac{1}{1-\alpha}}$. Let us denote this level of k as $\hat{k} \equiv (s\phi)^{\frac{1}{1-\alpha}}$.

The kk locus reaches its maximum when $s\phi\alpha k_t^{\alpha-1} - 1 = 0$, that is, when $k = (s\alpha\phi)^{\frac{1}{1-\alpha}}$. Let us denote this level of k by $\bar{k} \equiv (s\alpha\phi)^{\frac{1}{1-\alpha}}$.

The kk locus, starting from $(0,0)$, is increasing for $k_t < \bar{k}$ and decreasing for $k_t > \bar{k}$. Note that, for $k_t > \hat{k}$, only negative prices could sustain a positive capital intensity.

Let us now consider the pp locus, along which nursing home prices are constant. This is defined by the relation:

$$p_t = c + \frac{(\gamma\theta + \delta(1-\theta))}{\left[\theta(1-s) + \theta\mu s\phi\alpha\left[s\left(\phi k_t^{\alpha} - \lambda p_t\right)\right]^{\alpha-1} + (1-\theta)\mu\right]} L^2$$

That expression can be rewritten as:

$$\begin{aligned} & (p_t - c)\left[\theta(1-s) + \theta\mu s\phi\alpha\left[s\left(\phi k_t^{\alpha} - \lambda p_t\right)\right]^{\alpha-1} + (1-\theta)\mu\right] \\ = \; & (\gamma\theta + \delta(1-\theta))\, L^2 \end{aligned}$$

At $k_t = 0$, this expression is:

$$\begin{aligned} & (p_t - c)\left[\theta(1-s) + \theta\mu s^{\alpha}\phi\alpha\,(-\lambda)^{\alpha-1}\,(p_t)^{\alpha-1} + (1-\theta)\mu\right] \\ = \; & (\gamma\theta + \delta(1-\theta))\, L^2 \end{aligned}$$

Let us denote $\Omega \equiv \theta(1-s) + (1-\theta)\mu$ and $\Lambda \equiv (\gamma\theta + \delta(1-\theta))\, L^2$

Let us suppose that the solution of

$$(p_t - c)\left[\Omega + \theta\mu s^{\alpha}\phi\alpha\,(-\lambda)^{\alpha-1}\,(p_t)^{\alpha-1}\right] = \Lambda$$

is a strictly positive level of p_t denoted by $\bar{p} > 0$.

Under that assumption, the pp locus lies above the kk locus at $k_t = 0$.

Consider now the level of the pp locus when $k_t = \bar{k} \equiv (s\alpha\phi)^{\frac{1}{1-\alpha}}$. We have:

$$(p_t - c)\left[\Omega + \theta\mu s\phi\alpha\left[s\left(\phi\left(s\alpha\phi\right)^{\frac{\alpha}{1-\alpha}} - \lambda p_t\right)\right]^{\alpha-1}\right] = \Lambda$$

Let us suppose that there is a unique solution $p_t > 0$ and denote the solution of this expression by $\tilde{p}$.

Under the assumption that

$$\frac{s\phi\left(s\alpha\phi\right)^{\frac{\alpha}{1-\alpha}} - \left(s\alpha\phi\right)^{\frac{1}{1-\alpha}}}{\lambda s} > \tilde{p}$$

(obtained from equation (62)), we have that the kk locus lies above the pp locus at $k_t = \bar{k}$. Given that the kk locus lies below the pp locus at $k_t = 0$, we know, by continuity, that the two loci must intersect at least once for a value of $k_t \in \left]0, \left(s\alpha\phi\right)^{\frac{1}{1-\alpha}}\right[$.

Consider now the level of the pp locus when $k_t = \hat{k} \equiv (s\phi)^{\frac{1}{1-\alpha}}$. We have:

$$(p_t - c)\left[\Omega + \theta\mu s\phi\alpha\left[s\left(\phi\left(s\phi\right)^{\frac{\alpha}{1-\alpha}} - \lambda p_t\right)\right]^{\alpha-1}\right] = \Lambda$$

Let us suppose that there is a unique solution $p_t > 0$ and denote the solution of this expression by $\hat{p}$.

Given $\hat{p} > 0$, the pp locus lies above the kk locus at $k = \hat{k}$. Given that, at $k_t = (s\alpha\phi)^{\frac{1}{1-\alpha}}$, the pp locus lies below the kk locus, we know for sure, by continuity, that the kk locus and the pp locus intersect at least once for a value of $k_t \in \left]\left(s\alpha\phi\right)^{\frac{1}{1-\alpha}}, \left(s\phi\right)^{\frac{1}{1-\alpha}}\right[$.

Note that, at a stationary equilibrium, we have:

$$\begin{aligned} k &= s\left(\phi k^{\alpha} - \lambda p\right) \\ p &= c + \frac{\Lambda}{\left[\mu s\theta\phi\alpha\left[s\left(\phi k^{\alpha} - \lambda p\right)\right]^{\alpha-1} + \Omega\right]} = c + \frac{\Lambda}{\left[\mu s\theta\phi\alpha k^{\alpha-1} + \Omega\right]} \end{aligned}$$

From the second expression, it is clear that, when the equilibrium level of k is larger, the equilibrium level of p is larger as well.

Thus, if we denote the two stationary equilibria as (k_1, p_1) and (k_2, p_2), we have $k_1 < k_2$ and $p_1 < p_2$.

Replacing the second equation into the first one, the equilibrium capital level can be written as:

$$k = s\left(\phi k^{\alpha} - \lambda\left(c + \frac{\Lambda}{\left[\mu s\theta\phi\alpha k^{\alpha-1} + \Omega\right]}\right)\right)$$

Hence

$$k = \frac{s\phi k^{\alpha}\left[\mu s\theta\phi\alpha k^{\alpha-1} + \Omega\right] - \lambda cs\left[\mu s\theta\phi\alpha k^{\alpha-1} + \Omega\right] - s\lambda\Lambda}{\left[\mu s\theta\phi\alpha k^{\alpha-1} + \Omega\right]}$$

or equivalently,

$$\left[-\left(s\phi\right)^{2}\mu\theta\alpha\right]k^{2\alpha-1}+k\Omega+k^{\alpha}\left[\mu s\theta\phi\alpha-s\phi\Omega\right]+\lambda cs^{2}\mu\theta\phi\alpha k^{\alpha-1}+\lambda cs\Omega+\lambda s\Lambda=0 \tag{63}$$

The levels k_1 and k_2 are solutions of this equation.

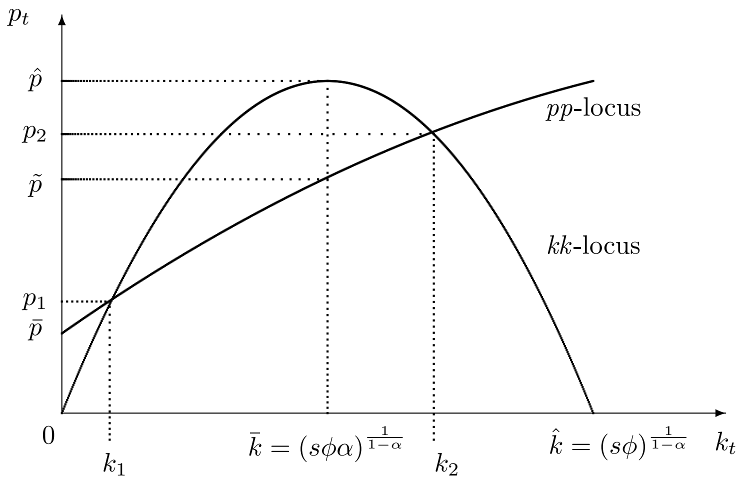

Figure 1: Existence of a stationary equilibrium.

9.3 Proof of Proposition 7

Let us now consider the stability of those stationary equilibria.

We have:

$$\begin{aligned} k_{t+1} &= s\left(\phi k_t^{\alpha}-\lambda p_t\right)\equiv F\left(k_t,p_t\right) \\ p_{t+1} &= c+\frac{\Lambda}{\left[\mu s\theta\phi\alpha\left[s\left(\phi k_t^{\alpha}-\lambda p_t\right)\right]^{\alpha-1}+\Omega\right]}\equiv G(k_t,p_t) \end{aligned}$$

The Jacobian matrix is:

$$J\equiv\begin{pmatrix} \frac{\partial F(k_t,p_t)}{\partial k_t} & \frac{\partial F(k_t,p_t)}{\partial p_t} \\ \frac{\partial G(k_t,p_t)}{\partial k_t} & \frac{\partial G(k_t,p_t)}{\partial p_t} \end{pmatrix}$$

We have:

$$\begin{aligned}
\frac{\partial F(k_t,p_t)}{\partial k_t} &= s\phi\alpha k_t^{\alpha-1} \\
\frac{\partial F(k_t,p_t)}{\partial p_t} &= -s\lambda \\
\frac{\partial G(k_t,p_t)}{\partial k_t} &= \frac{-\Lambda\left[\mu s^{\alpha}\theta\phi^2\alpha^2(\alpha-1)\left[\left(\phi k_t^{\alpha}-\lambda p_t\right)\right]^{\alpha-2}k_t^{\alpha-1}\right]}{\left[\mu s\theta\phi\alpha\left[s\left(\phi k_t^{\alpha}-\lambda p_t\right)\right]^{\alpha-1}+\Omega\right]^2} \\
\frac{\partial G(k_t,p_t)}{\partial p_t} &= \frac{\Lambda\left[\lambda\mu s^{\alpha}\theta\phi\alpha(\alpha-1)\left[\left(\phi k_t^{\alpha}-\lambda p_t\right)\right]^{\alpha-2}\right]}{\left[\mu s\theta\phi\alpha\left[s\left(\phi k_t^{\alpha}-\lambda p_t\right)\right]^{\alpha-1}+\Omega\right]^2}
\end{aligned}$$

The determinant of the Jacobian matrix is:

$$\begin{aligned}
& s\phi\alpha k_t^{\alpha-1}\frac{\Lambda\left[\lambda\mu s^{\alpha}\theta\phi\alpha(\alpha-1)\left[\left(\phi k_t^{\alpha}-\lambda p_t\right)\right]^{\alpha-2}\right]}{\left[\mu s\theta\phi\alpha\left[s\left(\phi k_t^{\alpha}-\lambda p_t\right)\right]^{\alpha-1}+\Omega\right]^2} \\
& -(-\lambda s)\frac{-\Lambda\left[\mu s^{\alpha}\theta\phi^2\alpha^2(\alpha-1)\left[\left(\phi k_t^{\alpha}-\lambda p_t\right)\right]^{\alpha-2}k_t^{\alpha-1}\right]}{\left[\mu s\theta\phi\alpha\left[s\left(\phi k_t^{\alpha}-\lambda p_t\right)\right]^{\alpha-1}+\Omega\right]^2} \\
= \; & 0
\end{aligned}$$

The trace is:

$$s\phi\alpha k_t^{\alpha-1}+\frac{\Lambda\left[\lambda\mu s^{\alpha}\theta\phi\alpha(\alpha-1)\left[\left(\phi k_t^{\alpha}-\lambda p_t\right)\right]^{\alpha-2}\right]}{\left[\mu s\theta\phi\alpha\left[s\left(\phi k_t^{\alpha}-\lambda p_t\right)\right]^{\alpha-1}+\Omega\right]^2}$$

At the stationary equilibrium, this can be reduced to:

$$s\phi\alpha k^{\alpha-1}+\frac{\Lambda\lambda\mu s^{\alpha}\theta\phi\alpha(\alpha-1)k^{\alpha-2}}{\left[\mu s^{\alpha}\theta\phi\alpha k^{\alpha-1}+\Omega\right]^2}$$

The Jacobian matrix has two eigen values. One is zero and the other is

$$s\phi\alpha k^{\alpha-1}+\frac{\Lambda\lambda\mu s^{\alpha}\theta\phi\alpha(\alpha-1)k^{\alpha-2}}{\left[\mu s^{\alpha}\theta\phi\alpha k^{\alpha-1}+\Omega\right]^2}$$

Hence, two cases can arise:

- If
$$\left|s\phi\alpha k^{\alpha-1}+\frac{\Lambda\lambda\mu s^{\alpha}\theta\phi\alpha(\alpha-1)k^{\alpha-2}}{\left[\mu s^{\alpha}\theta\phi\alpha k^{\alpha-1}+\Omega\right]^2}\right|<1$$
  then the stationary equilibrium is locally stable.

- If
$$\left| s\phi\alpha k^{\alpha-1} + \frac{\Lambda\lambda\mu s^{\alpha}\theta\phi\alpha\left(\alpha-1\right)k^{\alpha-2}}{\left[\mu s^{\alpha}\theta\phi\alpha k^{\alpha-1}+\Omega\right]^{2}} \right| > 1$$
then the stationary equilibrium is a saddle point (and thus unstable).

To go further in the investigation, we need to be able to have closed form solutions for equilibrium levels of k and p. For that purpose, let us suppose that $\alpha = \frac{1}{2}$ and $c = 0$.

Using (63), we have, at the equilibrium, that k is a solution to

$$-\frac{\left(s\phi\right)^{2}\mu\theta}{2} + k\Omega + k^{\frac{1}{2}}\left[\frac{\mu s\theta\phi}{2} - s\phi\Omega\right] + \lambda s\Lambda = 0$$

Denoting $x \equiv k^{1/2}$, this condition can be written as:

$$\Omega x^{2} + x\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right) + \lambda s\Lambda - \frac{\left(s\phi\right)^{2}\mu\theta}{2} = 0.$$

We have

$$\Delta = \left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right)^{2} - 4\Omega\left(\lambda s\Lambda - \frac{\left(s\phi\right)^{2}\mu\theta}{2}\right)$$

and thus, the solutions to the above polynomial are:

$$x_{1,2} = \frac{-\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right) \pm \sqrt[2]{\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right)^{2} - 4\Omega\left(\lambda s\Lambda - \frac{(s\phi)^{2}\mu\theta}{2}\right)}}{2\Omega}$$

implying

$$k_{1,2} = \left[\frac{-\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right) \pm \sqrt[2]{\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right)^{2} - 4\Omega\left(\lambda s\Lambda - \frac{(s\phi)^{2}\mu\theta}{2}\right)}}{2\Omega}\right]^{2}.$$

Hence prices satisfy

$$p_{1,2} = c + \frac{\Lambda}{\left[\mu s\theta\phi\alpha k_{1,2}^{\alpha-1} + \Omega\right]}$$

In that case, the stability condition becomes, in case of k_1:

$$\left| s\phi\frac{1}{2}\left[\frac{-\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)+\sqrt[2]{\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)^2-4\Omega\left(\lambda s\Lambda-\frac{(s\phi)^2\mu\theta}{2}\right)}}{2\Omega}\right]^{-1} -\frac{1}{4}\frac{\Lambda\mu s^2\theta\phi\left[\frac{-\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)+\sqrt[2]{\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)^2-4\Omega\left(\lambda s\Lambda-\frac{(s\phi)^2\mu\theta}{2}\right)}}{2\Omega}\right]^{-3}}{\left[\frac{\mu s^{1/2}\theta\phi\left[\frac{-\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)+\sqrt[2]{\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)^2-4\Omega\left(\lambda s\Lambda-\frac{(s\phi)^2\mu\theta}{2}\right)}}{2\Omega}\right]^{-1}}{2}+\Omega\right]^2} \right| < 1$$

or, alternatively:

$$\left| \frac{\Omega s\phi}{-\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)+\Psi} - \frac{1}{4}\frac{\Lambda\mu s^2\theta\phi\left[\frac{2\Omega}{-\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)+\Psi}\right]^3}{\left[\frac{\Omega\mu s^{1/2}\theta\phi}{-\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)+\Psi}+\Omega\right]^2} \right| < 1$$

In the case of k_2, the condition for stability is:

$$\left| \frac{\Omega s\phi}{-\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)-\Psi} - \frac{1}{4}\frac{\Lambda\mu s^2\theta\phi\left[\frac{2\Omega}{-\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)-\Psi}\right]^3}{\left[\frac{\Omega\mu s^{1/2}\theta\phi}{-\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)-\Psi}+\Omega\right]^2} \right| < 1$$

where $\Psi \equiv \sqrt[2]{\left(\frac{\mu s\theta\phi}{2}-s\phi\Omega\right)^2-4\Omega\left(\lambda s\Lambda-\frac{(s\phi)^2\mu\theta}{2}\right)}$